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POSITION ERROR PROPAGATION IN THE SIMPLEX
STRAPDOWN NAVIGATION SYSTEM

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1.0 INTRODUCTION

This report documents the results of an analysis of the effects of deterministic error sources on position error in the simplex strapdown navigation system. Improving the long term accuracy of the system was addressed in two phases: understanding and controlling the error within the system, and defining methods of damping the net system error through the use of an external reference velocity or position. The analysis of error propagation is an initial step in controlling the error in an undamped system.

Review of the flight and ground data revealed error containing the Schuler frequency as well as non-repeatable trends. The Schuler frequency indicates the presence of error components which can be modeled by deterministic sources such as initial misalignment of sensor biases. Small initialization and instrument errors are magnified by their exciting sinusoidal error oscillations with periods of 84 minutes, the Schuler period. Initial level alignment errors and accelerometer bias errors produce bounded sinusoidally varying position error components⁽¹⁾. The only unbounded terms are those involving gyro bias and azimuth error coupled with velocity. All forms of Schuler-periodic position error were found to be sufficiently large, to require update or damping capability unless the source coefficients can be limited to values less than those used in this analysis for misalignment and gyro and accelerometer bias.

The first-order effects of the deterministic error sources were determined with a simple error propagator which provided plots of error time functions in response to various source error values.

⁽¹⁾Leondes, Guidance and Navigation of Aerospace Vehicles, p. 145

2.0 ERROR PROPAGATOR CHARACTERISTICS

The propagator demonstrated position error propagation as a function of time within the simplex system operating environment constraints: the period of interest was on the order of two hours, and the speeds during the test runs were less than 100 knots. The trajectory over which the errors were propagated was horizontal at a constant heading and speed⁽²⁾ specified in the input data for a period of 95 minutes. The error sources modeled included initial level and azimuth misalignment, accelerometer constant bias and scale factor error and gyroscope constant bias.

Because of the short time period of interest, cross-coupling, earth-rate coupling and Coriolis-correction term errors could be ignored without significantly changing the error shapes and magnitudes⁽³⁾. The assumption of no correlation between accelerometer bias and initial platform tilt was valid because the system was aligned either optically or off-line. That is, when self-alignment was employed, the alignment angles were passed to the navigation routines via the test operator. If self-alignment were included in the computer program so that navigation could commence without interruption following alignment, the error propagation from the accelerometer constant bias would be cancelled by that from an initial tilt in the reference.

The error propagation equations shown in Table 1 are derived⁽⁴⁾ for a locally level system, and include the transformation matrix components where they are needed to relate the sensor error sources from the triad to the level coordinate system.

3.0 ERROR CHARACTERISTICS

Although the propagator was designed to yield the net error resulting from any set of one or more source errors, the source errors were propagated individually to allow visibility of the relative significance of the sources.

⁽²⁾ *ibid.*; p. 143

⁽³⁾ Pitman, *Inertial Guidance*, p. 166

⁽⁴⁾ *ibid.*; pp. 169; Kayton, Fried, *Avionics Navigation Systems*, p. 325

The test-case values for each source were chosen to be within specification or the bounds of experience and to be a round number so that the test results could be easily scaled up or down in the evaluation of a subsequent case.

Roll and pitch initial misalignment test values were selected to be 30 arcseconds, representing the observed error for an optical alignment of a relatively smooth day. The least alignment error for an optimal optical alignment was approximately 8 arcseconds.

Azimuth misalignment was selected to be 30 arcseconds.

The test case value for accelerometer bias was $5 \times 10^{-5} \text{G}$. Bias stability was specified to be $1.5 \times 10^{-5} \text{G/}^{\circ}\text{F}$ and the temperature sensitivity of the bias was $1.0 \times 10^{-5} \text{G/}^{\circ}\text{F}$. The test value represented the effect of a 5°F temperature change at the instrument during the period following calibration. The assumption of a 5°F temperature change on the sensing element may or may not be representative of flight performance. Although the ambient compartment air temperature, which conditioned the sensor block, did fluctuate more than five degrees, sensor block temperature was not instrumented.

Accelerometer scale factor error was tested at its specification limit of 60 ppm.

Gyro bias test values were set at $0.01^{\circ}/\text{hr.}$, the one-sigma (1σ) random drift. A temperature change of 5°F would have caused a bias error of $.05^{\circ}/\text{hr.}$ and would have quintupled the error generated in response to the bias.

4.0 PROPAGATED ERROR

The error summary, Table 2, shows the error resulting from each source at various true headings. The supporting plots showing the shapes of the first order approximations are included as Figures 1 through 32. Because the strapdown was mechanically constrained to vehicle azimuth, the east and north position errors driven by a given source varied with azimuth. The

root-sum square (r.s.s.), or resultant, errors at peaks and ends of runs were relatively independent of azimuth. Because the accelerometers and gyros were mounted in the triad, one hardware error source propagated along all three axes in the L coordinate system. Only X and Y errors were calculated. The r.s.s. error from a particular hardware source was independent of azimuth or position in the triad. Care should be exercised when assessing the error caused by more than one gyro bias or more than one accelerometer bias. The resultant position error must be determined after component errors are derived for the source error combination.

The most significant error source was gyro constant bias. A single-instrument one-sigma bias error, $0.01^{\circ}/\text{hr.}$ produced an average r.s.s. error of approximately 4200 ft. (about 0.7 n.mi.). If the sensor temperature were five degrees different from its value during calibration and alignment and the gyro temperature sensitivity were as specified, the single-instrument bias contribution to resultant position error would be 3.5 nautical miles.

If all gyro bias errors were equal and if all had the same sign, position error would be small, on the order of 400 feet for the cases shown. (Had the gyros been mounted in a way other than simplex-system triad, resultant error could have been quite different.) If they were equal and one had an opposite sign, the resultant position error contribution would be increased to approximately 8500 feet (about 1.4 n.mi.), double the single-instrument contribution for one-sigma constant bias. This latter error would probably be increased by a temperature change but the assumption of quintupling is unwarranted because of multiple instrument involvement.

Accelerometer bias and initial level misalignment produce correlated bounded error with sinusoidal maxima at 42 minutes, the Schuler half-period. The average single-instrument selected accelerometer constant bias values was approximately 1710 feet (about 0.3 n.mi.). If temperature were not a factor, the specification bias stability could cause as much as 640 feet of resultant position error. If, as would be more representative of actual operation, more than one accelerometer bias error existed, the resultant error would be affected similarly to the way in which gyro-bias

resultant error varied. If all accelerometer bias were equal with the same sign, the resultant error would be about 3% of the resultant produced by a single-instrument bias error; if they were equal but one had the sign opposite that of the other two, the resultant error would be double the single value.

The effect of misalignment is not as complex because the triad is not directly involved - the propagation is somewhat more pure. The maximum resultant position error caused by misalignment about pitch or about roll was approximately 6090 feet (about 1.0 n.mi.). If roll and pitch misalignment or levelling error were both present, the effect would add component-wise to affect the change in resultant position error. If, in that hypothetical case, roll error were controllable to 15 arcseconds, and pitch error remained about 30 arcseconds, the resultant error would be about 12% greater than that from a single misalignment. Opposite signs would change the component error but not the resultant magnitude.

Most of the cases were initialized with the sensor-cube known off-level angles, Beta (β) and Theta (θ), at zero. Two cases with non-zero initial level angles verified that the error propagation caused by gyro bias was affected by the initial tilt of the triad.

Azimuth error and scale factor error are insignificant for the cases executed because of the relatively low speed of 80 knots.

The plots are included as supportive and reference material. They are keyed to the Summary Table (Table 2) by Run Number.

5.0 SUMMARY

The significant deterministic error sources in the simplex strapdown navigation system are initial misalignment, gyro bias and accelerometer bias.

With the error coefficients selected for analysis, the system could accumulate several miles of position error in one hour. The coefficients during the flight tests could have been larger than those selected.

Self-alignment with instantaneous transfer to the navigation mode can reliably reduce initial misalignments to the range of 4-7 arcseconds with a subsequent reduction by an order of magnitude in the error propagated. In addition, the level errors would then tend to be correlated with accelerometer bias, so that one cancelled the other.

Gyro bias error is the most significant contributor of those identified. The effect of temperature change on gyro bias is known. The actual temperature value and variations for the test runs are not known. It has been shown that a modest change of the sensor temperature can produce significant error. Temperature should, at least, be monitored during developmental system test. Laboratory tests should also be devised to precisely map the reaction of gyros to temperature. If significant reaction is determined, and significant temperature changes are detected in system test, software compensation for real-time temperature change should be provided.

The position error propagated from gyro bias equivalent to the one-sigma random drift value cannot be compensated within the undamped system. Additionally, the presence of stationary zero-mean noise in the gyro signal causes unbounded system error⁽⁵⁾. The system must be damped with an external reference velocity or position to provide satisfactory long-term accuracy. The results of this deterministic-error propagation can be used in refining the undamped system to simplify the damping requirement and process.

⁽⁵⁾ Leondes, p. 146

Table 1. Position Error Propagation Equations

Initial Level Misalignment ($\Delta\psi_X, \Delta\psi_Y$)

$$\Delta X = R_0 (\Delta\psi_Y \sin Az + \Delta\psi_X \cos Az) (1 - \cos \omega_S t)$$

$$\Delta Y = R_0 (\Delta\psi_X \cos Az - \Delta\psi_Y \sin Az) (1 - \cos \omega_S t)$$

Initial Azimuth Misalignment ($\Delta\psi_Z$)

$$\Delta X = \Delta\psi_Z V t \cos Az$$

$$\Delta Y = \Delta\psi_Z V t \sin Az$$

Accelerometer Bias Error ($\Delta B_X, \Delta B_Y, \Delta B_Z$)

$$\Delta X = (\Delta B_X M_{SL11} + \Delta B_Y M_{SL21} + \Delta B_Z M_{SL31}) \frac{(1 - \cos \omega_S t)}{\omega_S^2}$$

$$\Delta Y = (\Delta B_X M_{SL12} + \Delta B_Y M_{SL22} + \Delta B_Z M_{SL32}) \frac{(1 - \cos \omega_S t)}{\omega_S^2}$$

Accelerometer Scale Factor Error ($\Delta K_X, \Delta K_Y, \Delta K_Z$)

$$\begin{aligned} \Delta X = & - \left[\Delta K_X (M_{SL11}^2 V \sin Az + M_{SL11} M_{SL12} V \cos Az) \right. \\ & + \Delta K_Y (M_{SL21}^2 V \sin Az + M_{SL21} M_{SL22} V \cos Az) \\ & \left. + \Delta K_Z (M_{SL31}^2 V \sin Az + M_{SL31} M_{SL32} V \cos Az) \right] \left[t - \frac{\sin \omega_S t}{\omega_S} \right] \end{aligned}$$

Table 1. Position Error Propagation Equations (Continued)

$$\begin{aligned} \Delta Y = & - \left[\Delta K_X (M_{SL11} M_{SL12} V \sin Az + M_{SL12}^2 V \cos Az) \right. \\ & + \Delta K_Y (M_{SL21} M_{SL22} V \sin Az + M_{SL22}^2 V \cos Az) \\ & \left. + \Delta K_Z (M_{SL31} M_{SL32} V \sin Az + M_{SL32}^2 V \cos Az) \right] \left[t - \frac{\sin \omega_S t}{\omega_S} \right] \end{aligned}$$

Gyro Bias Error ($\Delta\phi_X, \Delta\phi_Y, \Delta\phi_Z$)

Level Components

$$\Delta X = R_0 (\Delta\phi_X M_{SL12} + \Delta\phi_Y M_{SL22} + \Delta\phi_Z M_{SL32}) \left(t - \frac{\sin \omega_S t}{\omega_S} \right)$$

$$\Delta Y = R_0 (\Delta\phi_X M_{SL11} + \Delta\phi_Y M_{SL21} + \Delta\phi_Z M_{SL31}) \left(t - \frac{\sin \omega_S t}{\omega_S} \right)$$

Azimuth Error Components

$$\Delta X = V \sin Az (\Delta\phi_X M_{SL13} + \Delta\phi_Y M_{SL23} + \Delta\phi_Z M_{SL33}) \left(t^2 - \frac{2(1 - \cos \omega_S t)}{\omega_S^2} \right)$$

$$\Delta Y = V \cos Az (\Delta\phi_X M_{SL13} + \Delta\phi_Y M_{SL23} + \Delta\phi_Z M_{SL33}) \left(t^2 - \frac{2(1 - \cos \omega_S t)}{\omega_S^2} \right)$$

Term Definitions

R_0	-	earth radius
ω_S	-	Schuler frequency (rad.)
V	-	Ground speed
Az	-	Azimuth
M_{SLij}	-	MSL matrix (as defined in Program Requirements Doc.)

Table 2. Propagator Error Summary

RUN NO.	AZ. (DEG)	δ (DEG)	θ (DEG)	TEND (MIN)	MISALIGN (ARSEC) X Y Z	ACC BIAS (X 10 ⁻⁵ G) X Y Z	ACC SF* (X 10 ⁻⁵) X Y Z	GYRO BIAS (X 10 ⁻² °/HR) X Y Z	X-TIME (MIN)	ΔX (FEET)	Y-TIME (MIN)	ΔY (FEET)	R.S.S. (FEET)
602	005	0	0	95	30				42	6062	42	-530	6085
603	005	0	0	95	30				42	530	42	6062	6085
604	005	0	0	95	30				95	112	95	10	112
605	005	0	0	95		5			42	152	42	1717	1724
606	005	0	0	95		5			42	-1543	42	-708	1698
607	005	0	0	95			5		42	1406	42	-963	1704
608	005	0	0	95			6		95	-2	95	-28	
609	005	0	0	95			6		95	-12	95	-6	
610	005	0	0	95			6		95	11	95	-8	
611	005	0	0	95				1	95	4280	95	497	4309
612	005	0	0	95				1	95	-1750	95	-3713	4105
613	005	0	0	95				1	95	-2384	95	3616	4331
621	050	0	0	95	30				42	3911	42	-4661	6084
622	050	0	0	95	30				42	4661	42	3911	6084
623	050	0	0	95	30				95	72	95	86	112
624	050	0	0	95		5			42	1322	42	1107	1724
625	050	0	0	95		5			42	-1592	42	590	1698
626	050	0	0	95			5		42	313	42	-1675	1704
627	050	0	0	95			6		95	-21	95	-18	
628	050	0	0	95			6		95	-13	95	5	
629	050	0	0	95			6		95	2	95	-13	
630	050	0	0	95				1	95	2483	95	3363	4180
631	050	0	0	95				1	95	1561	95	-3878	4180
632	050	0	0	95				1	95	-4071	95	856	4160
641	095	0	0	95	30				42	-530	42	-6062	6085
642	095	0	0	95		5			42	1717	42	-152	1724
643	095	0	0	95		5			42	-708	42	1543	1698
644	095	0	0	95			5		42	-963	42	-1406	1704
645	095	0	0	95			6		95	-28	95	2	
646	095	0	0	95				1	20	-299	95	4259	4267
647	095	0	0	95				1	95	3958	95	-1772	4337
648	095	0	0	95				1	95	-3373	95	-2405	4142
251	140	0	0	95				1	95	-3210	95	2660	4169
252	140	0	0	95				1	95	4036	95	1373	4263
253	140	0	0	95				1	95	-699	95	-4258	4314
261	185	0	0	95				1	95	-4280	95	-497	4309
262	185	0	0	95				1	95	1750	95	3713	4105
671	230	0	0	95				1	95	-2443	95	-3363	4180
633	050	5	-7	95	30				42	3911			
634	050	5	-7					1	95	2837	95	3765	4714

*Ground Speed = 30 Knots, all runs.

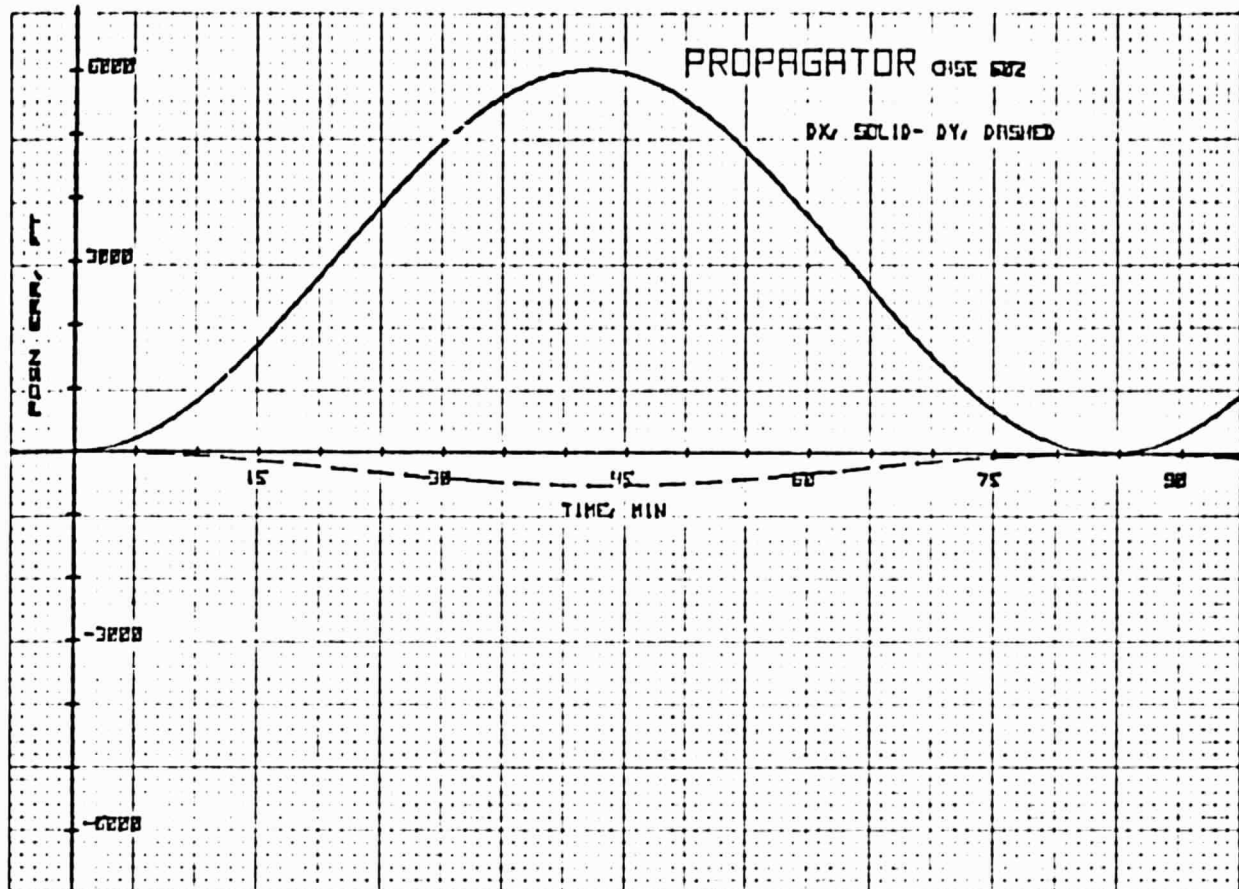
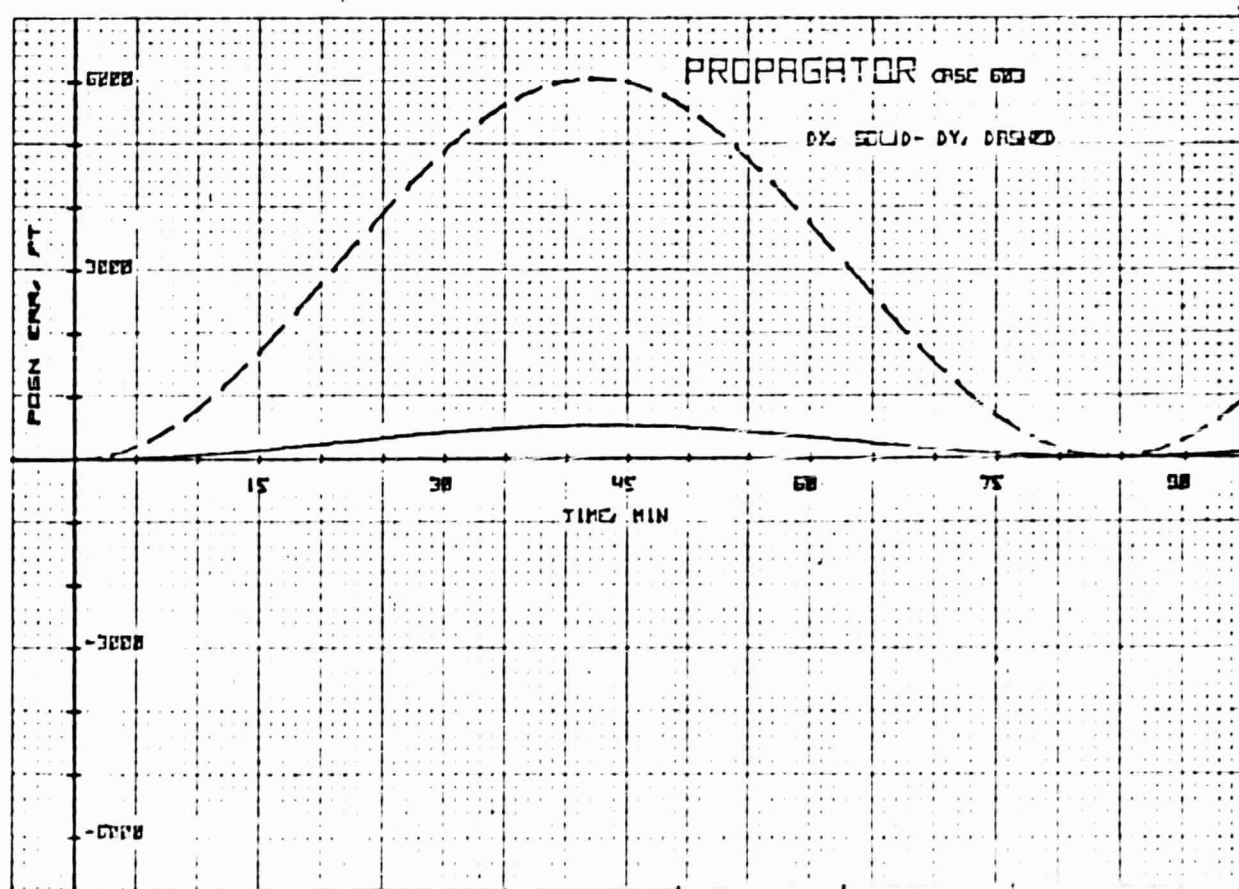


Figure 1. Case 602



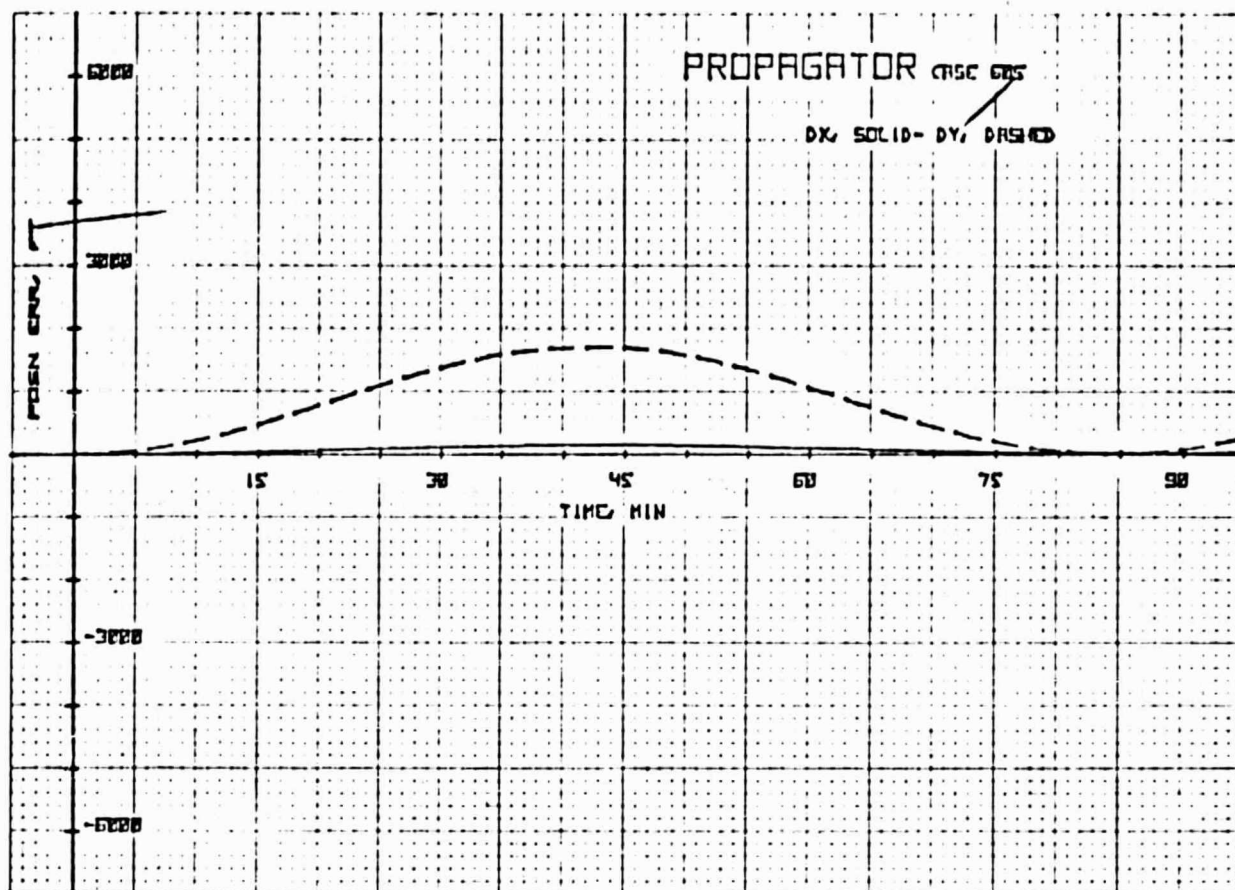
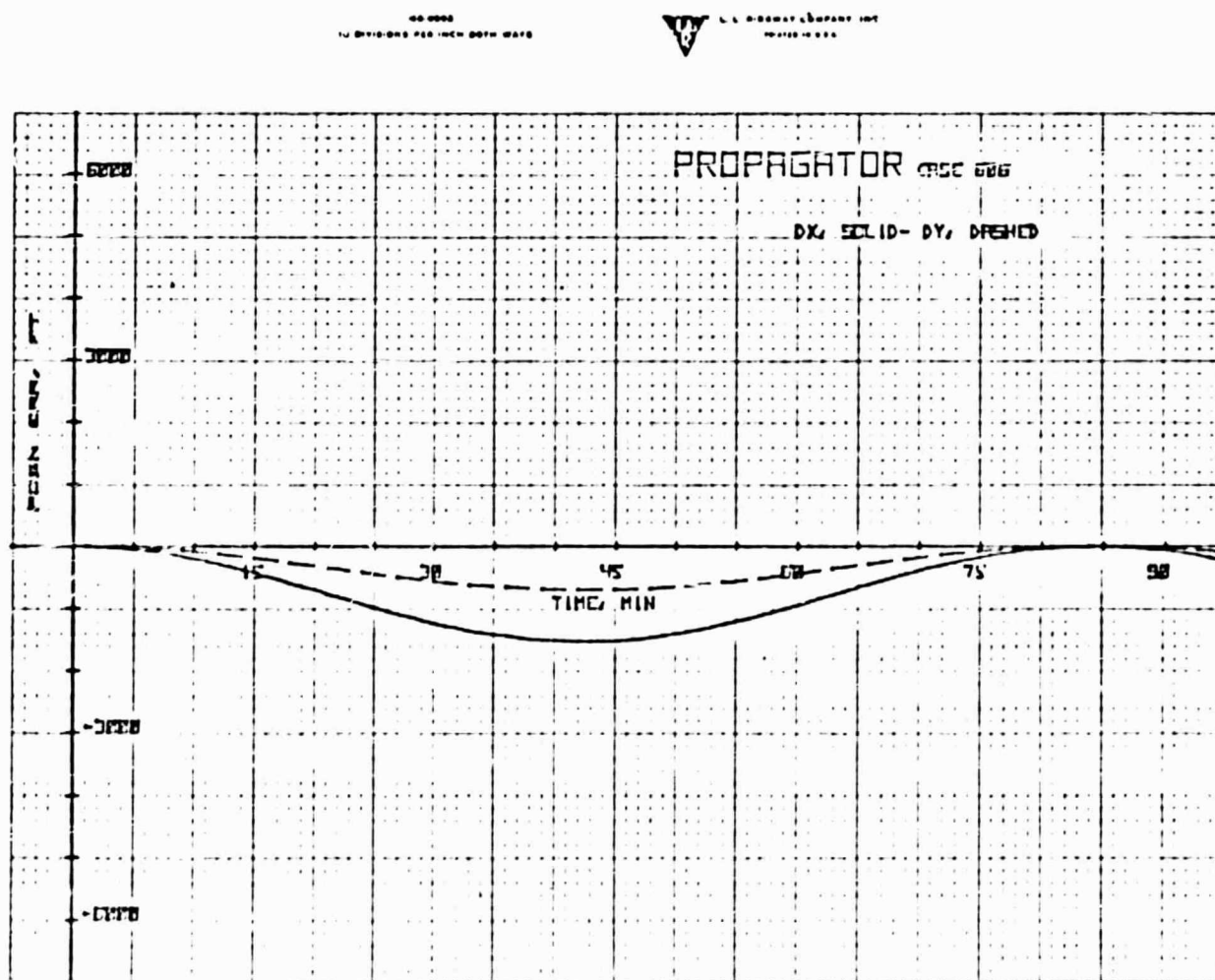


Figure 3. Case 605



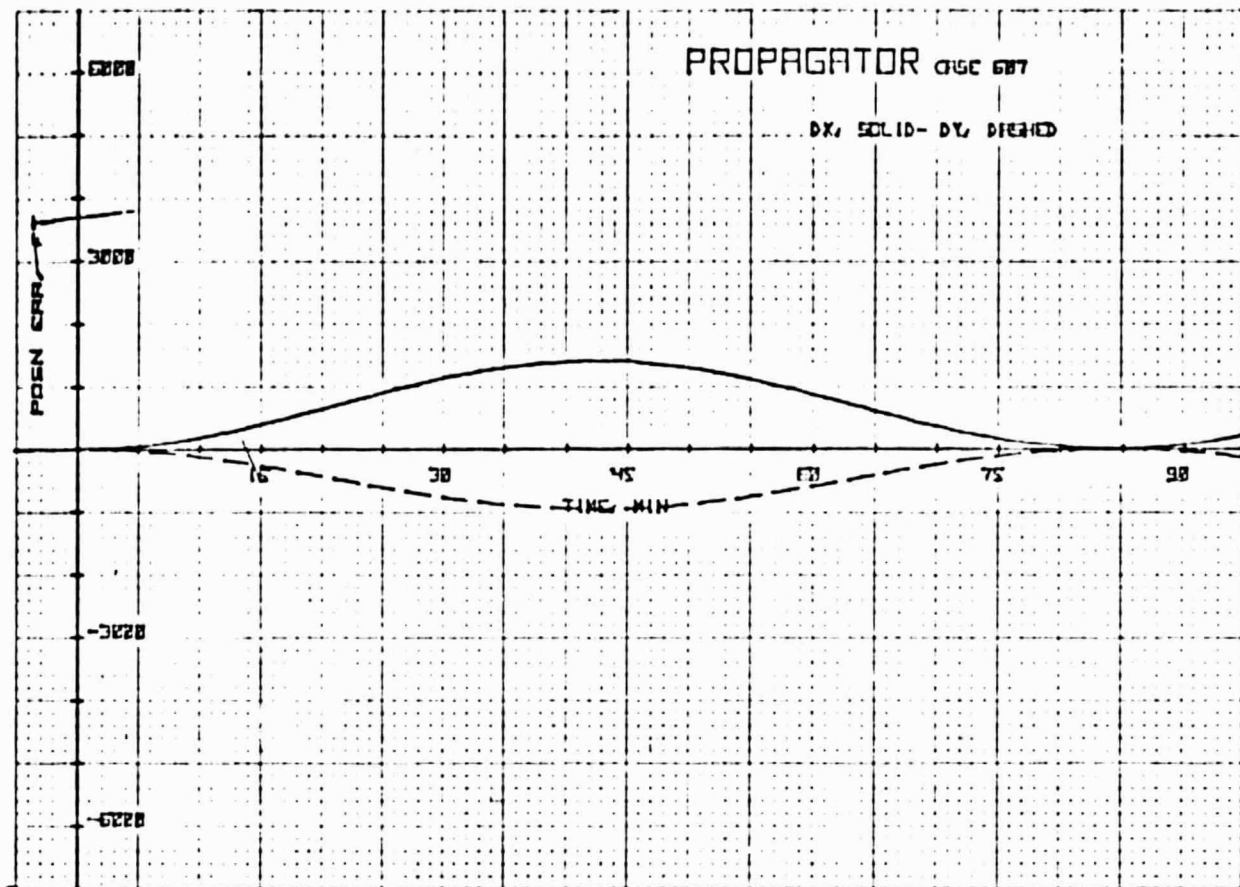
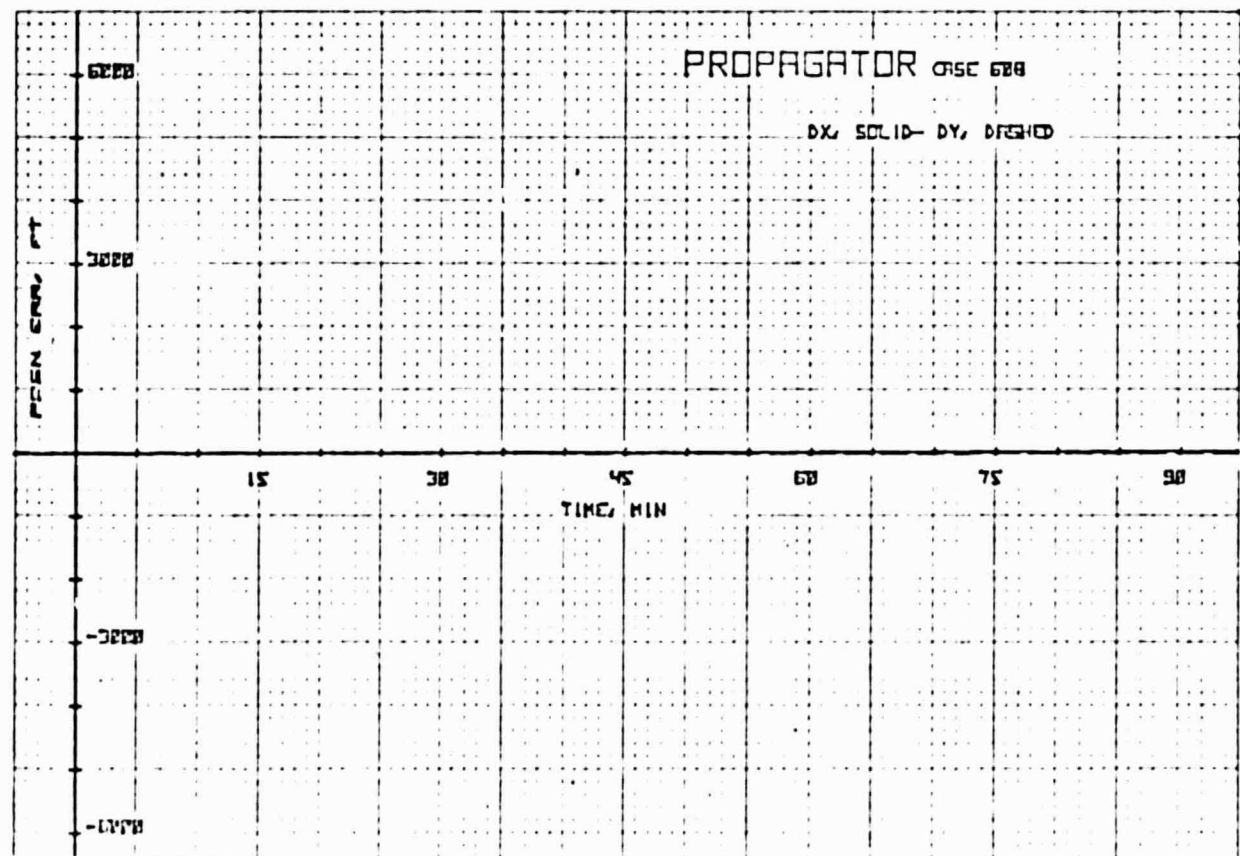


Figure 5. Case 607



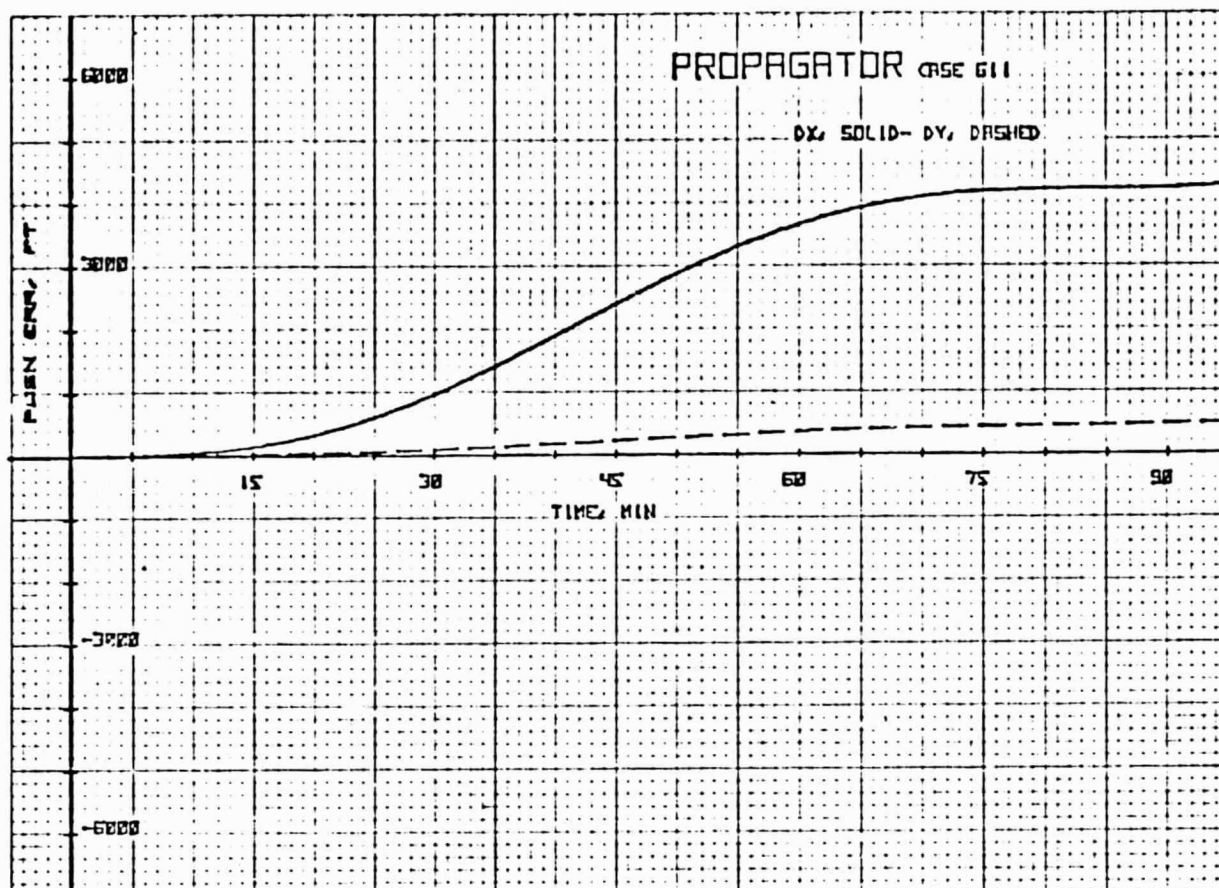


Figure 7. Case C11

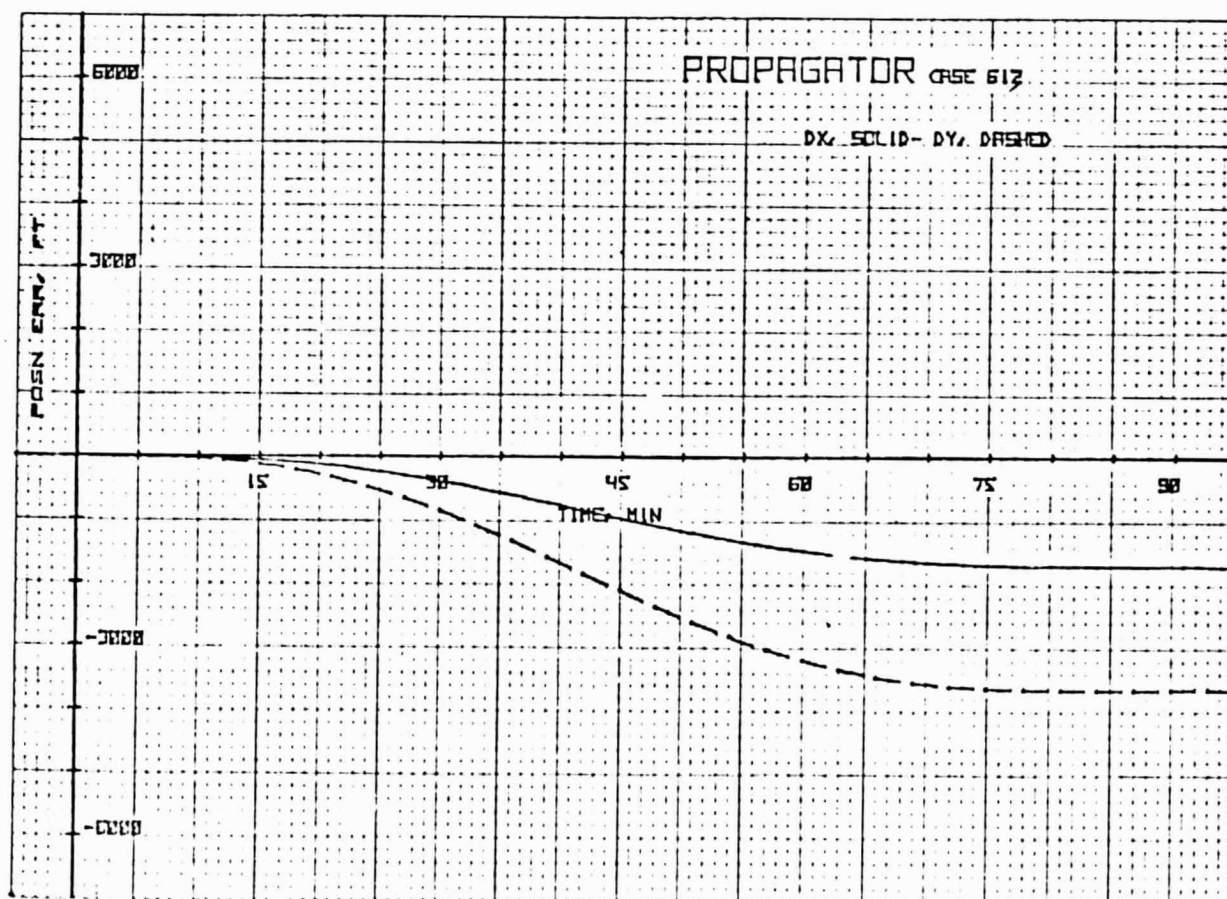


Figure 8. Case 612

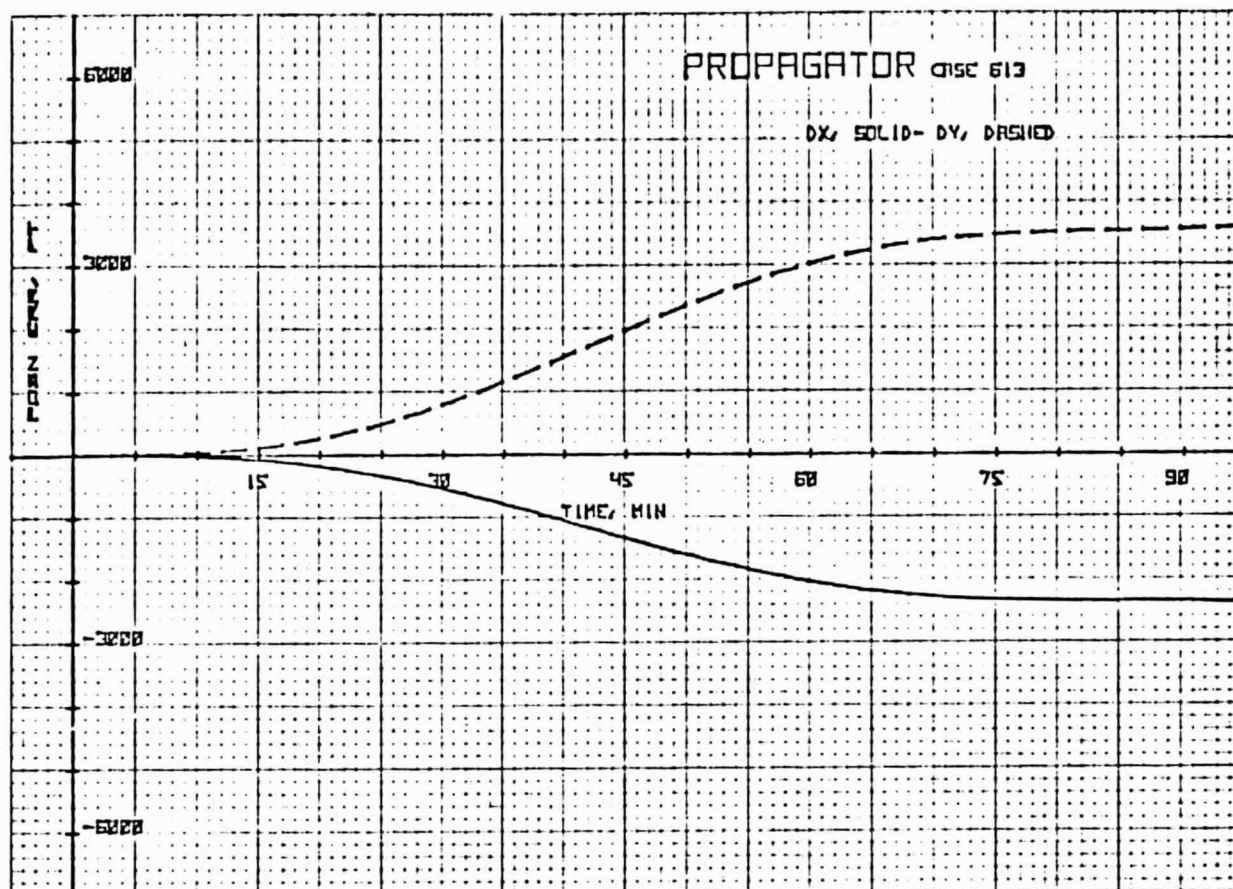


Figure 9. Case 613

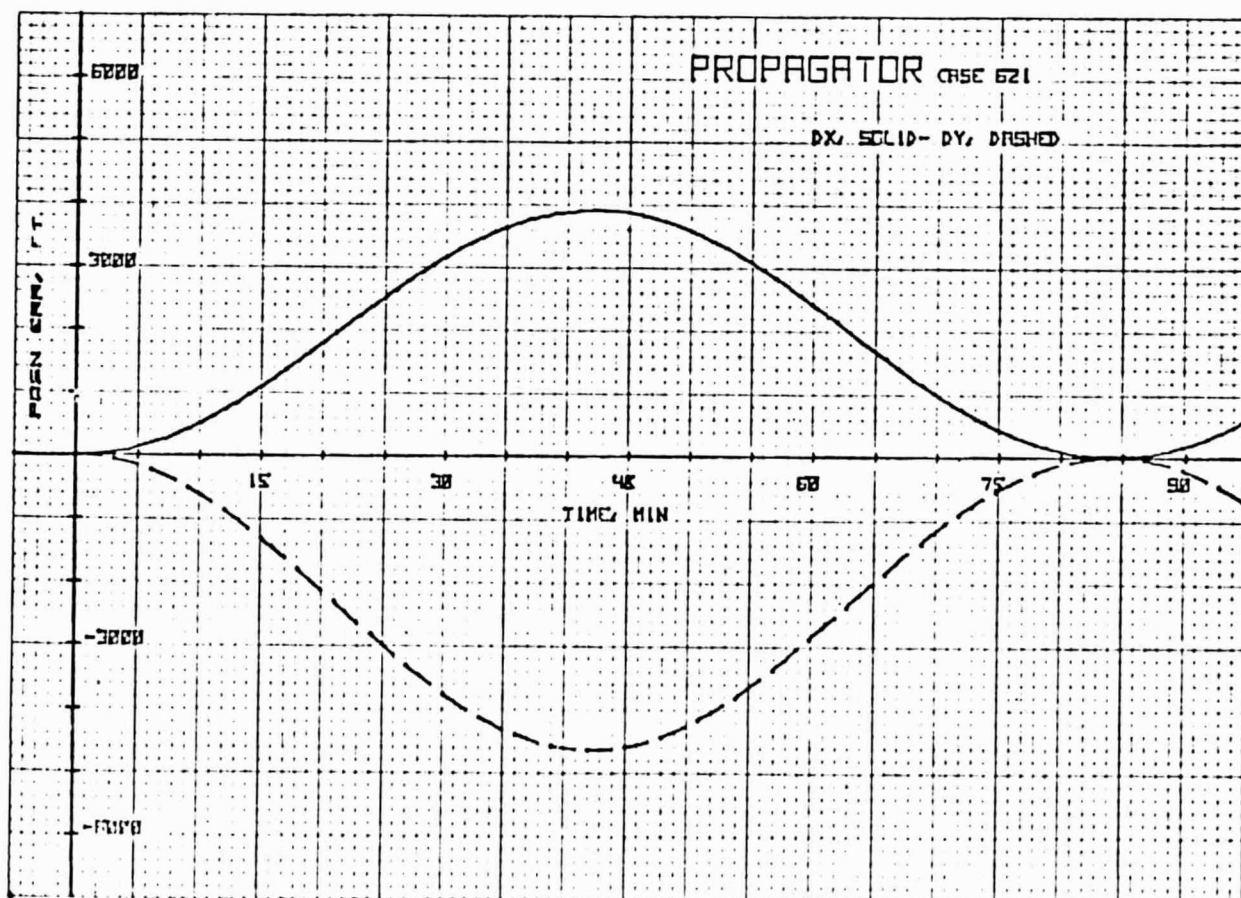


Figure 10. Case 621

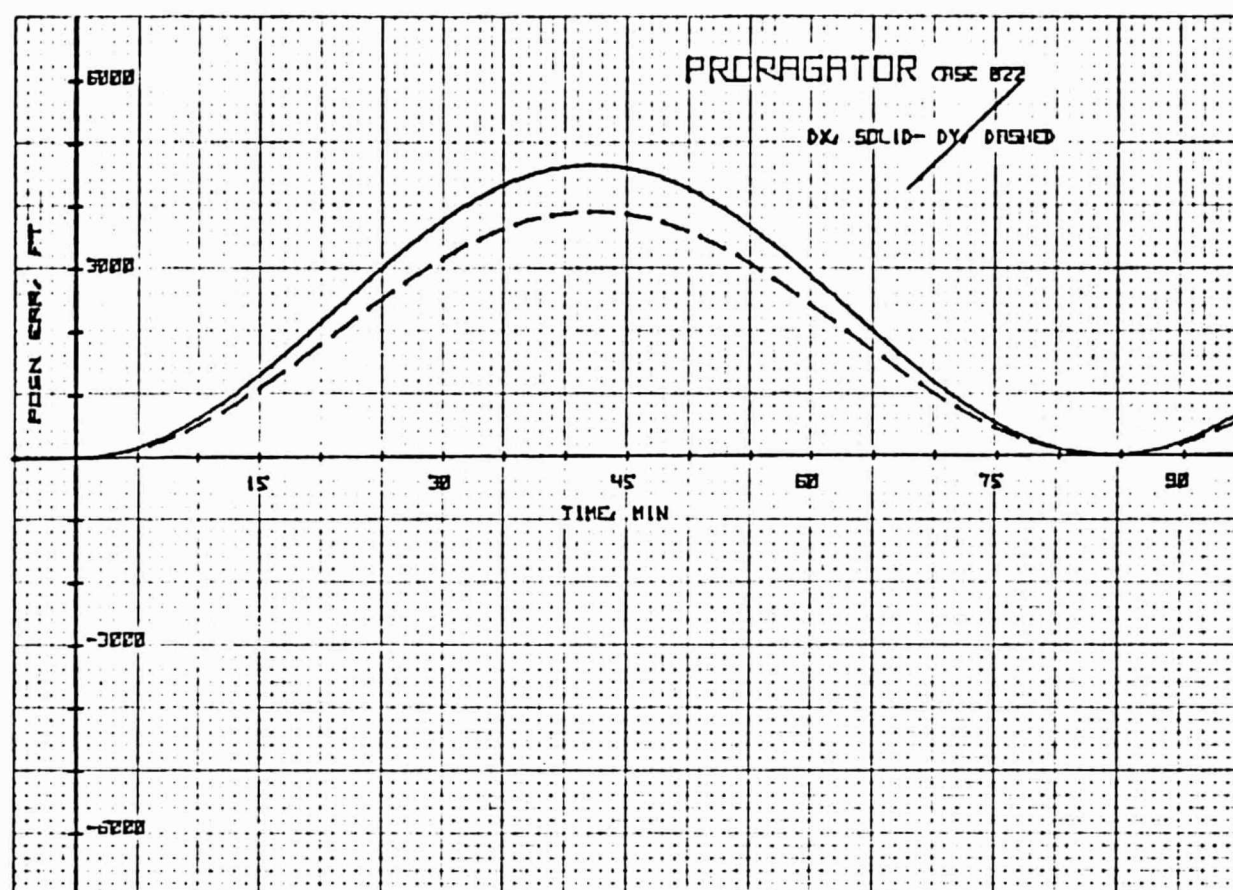


Figure 11. Case 622

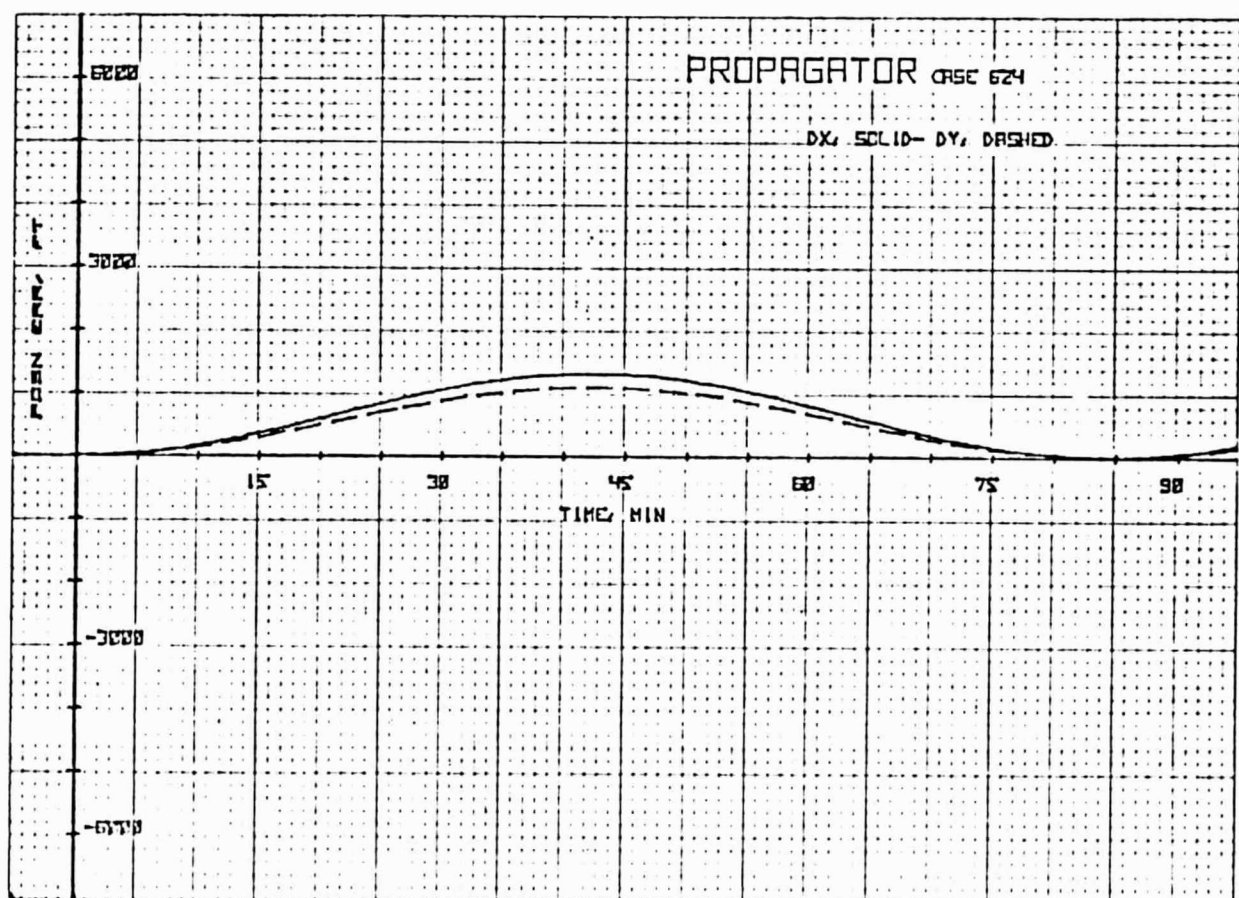


Figure 12. Case 624

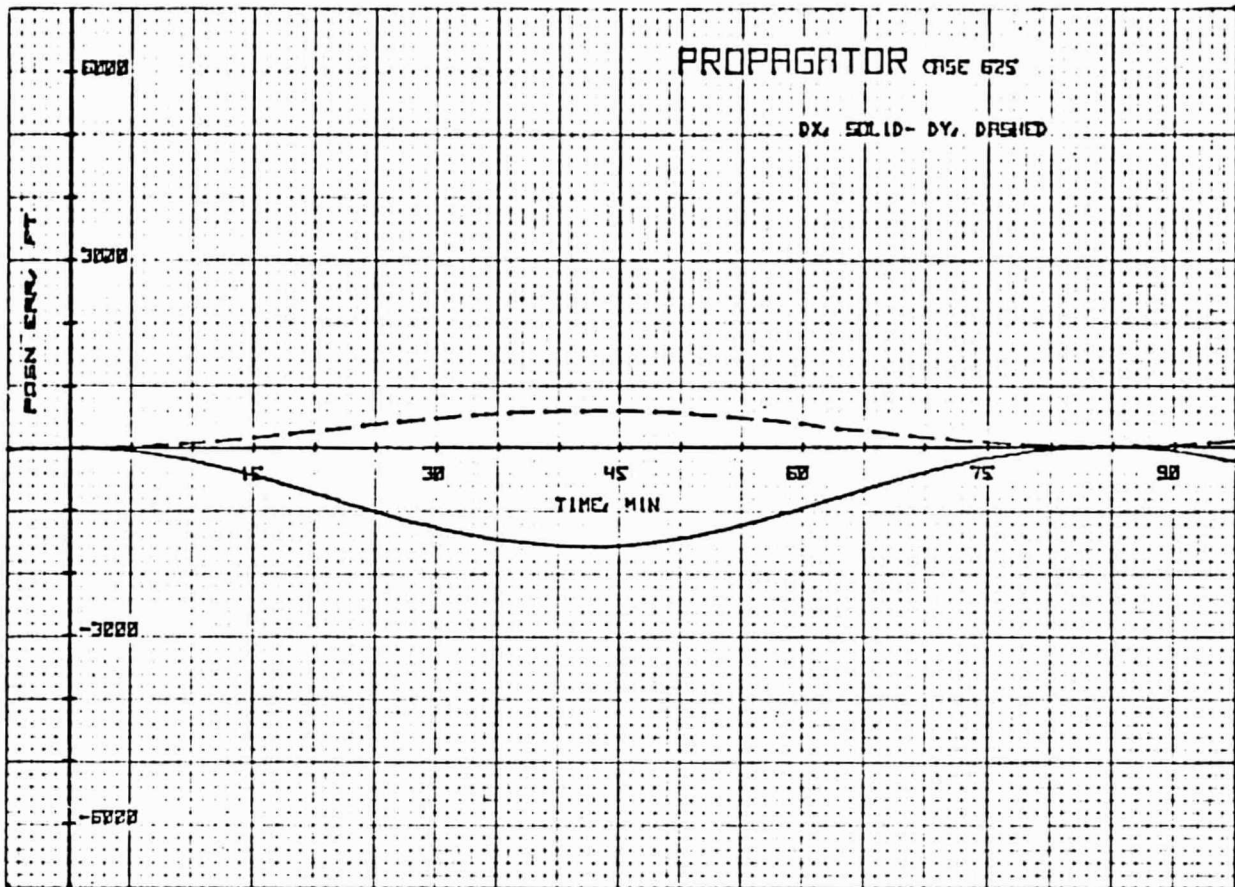


Figure 13. Case 625

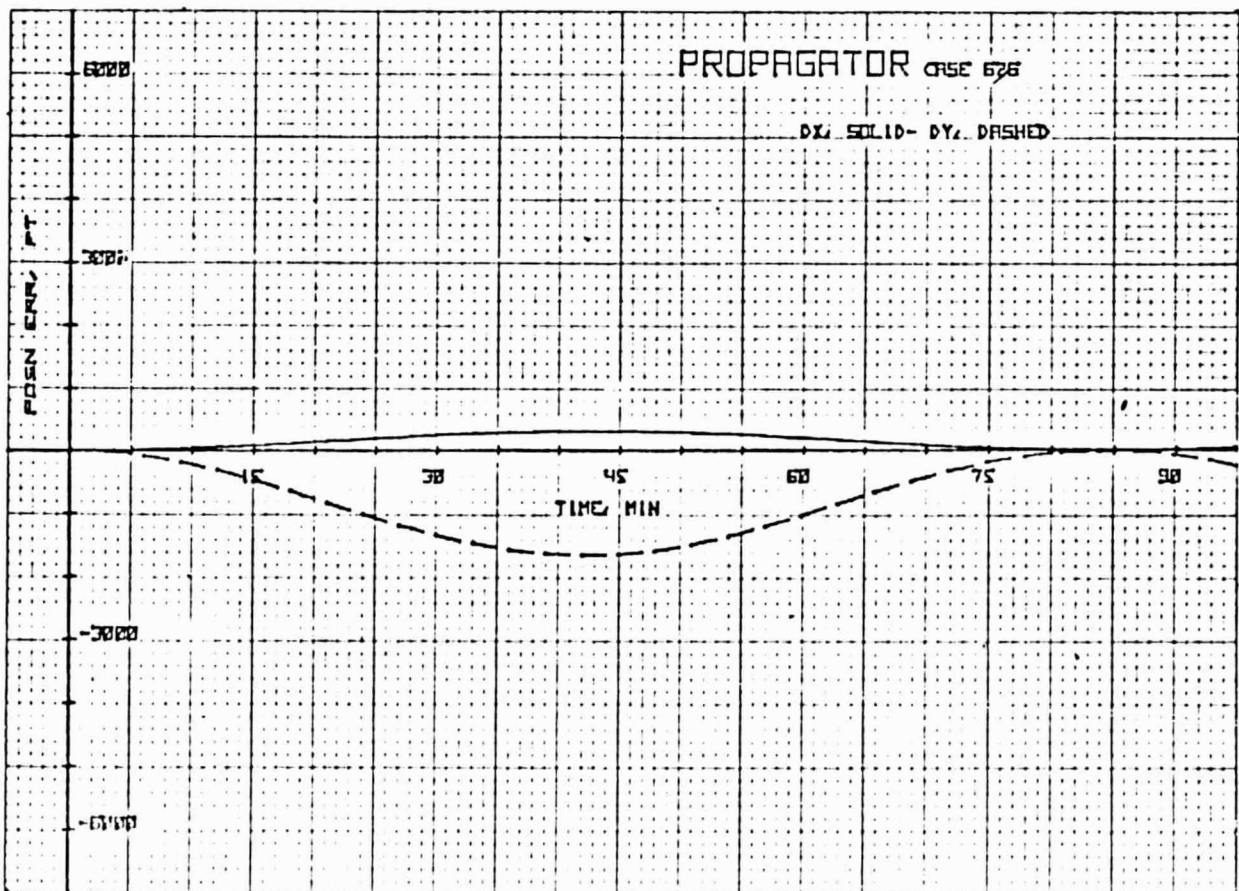


Figure 14. Case 626

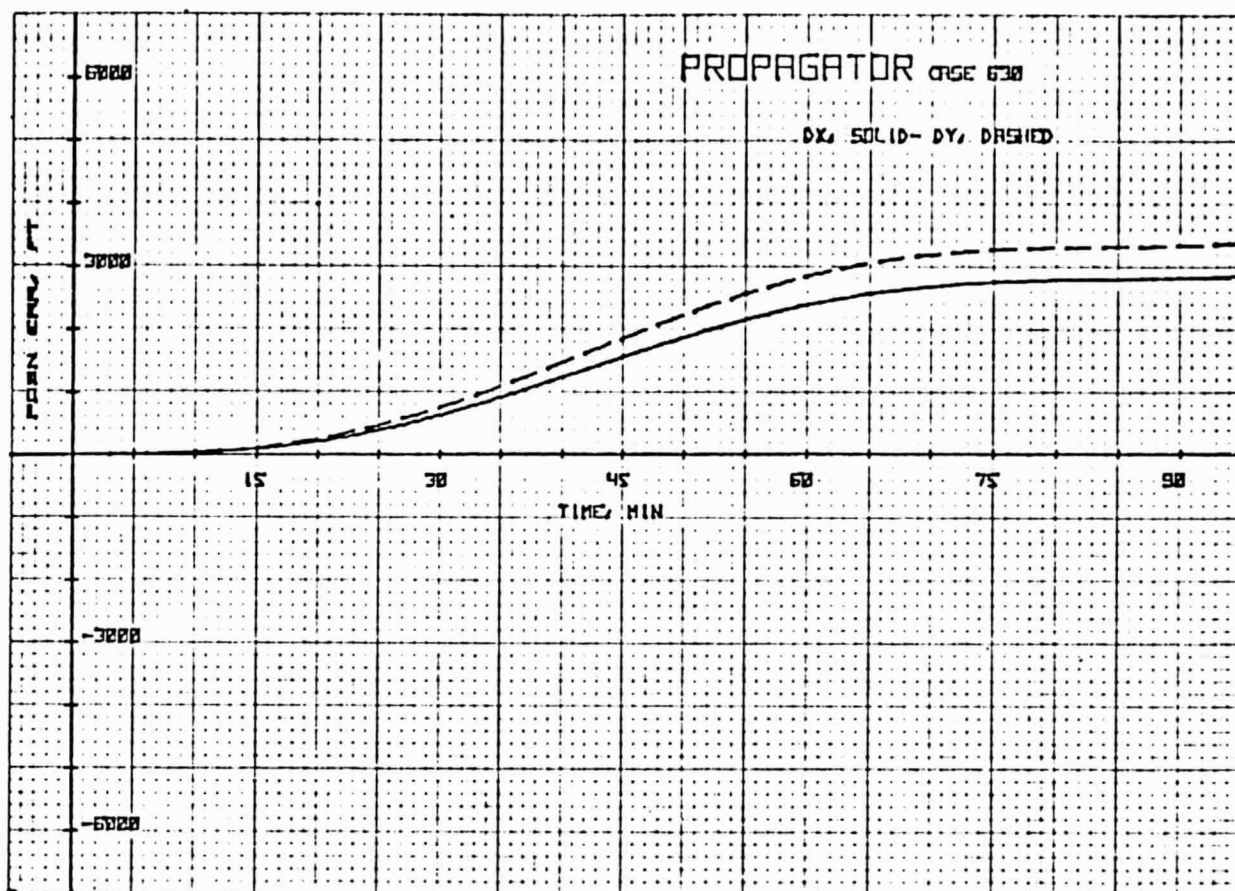
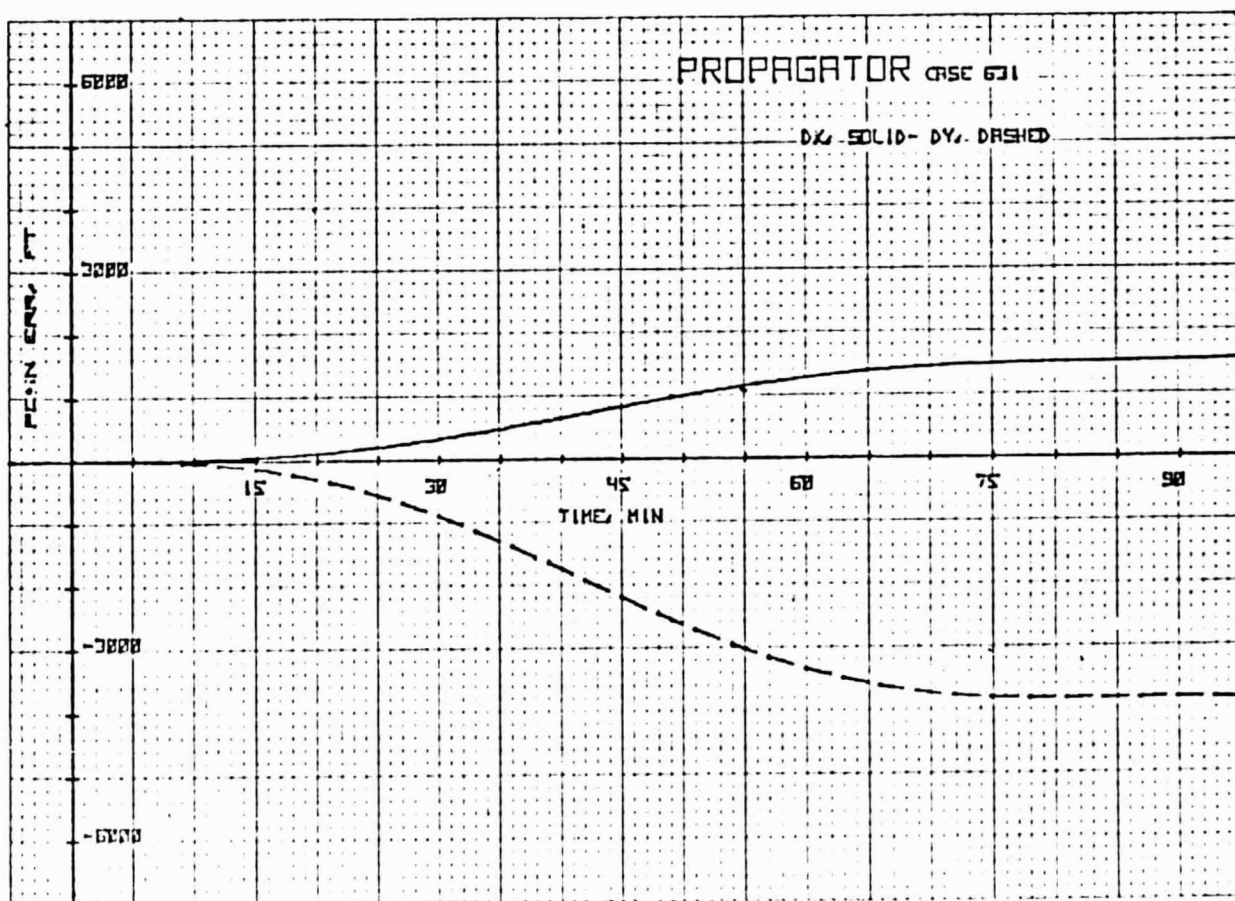


Figure 15. Case 630



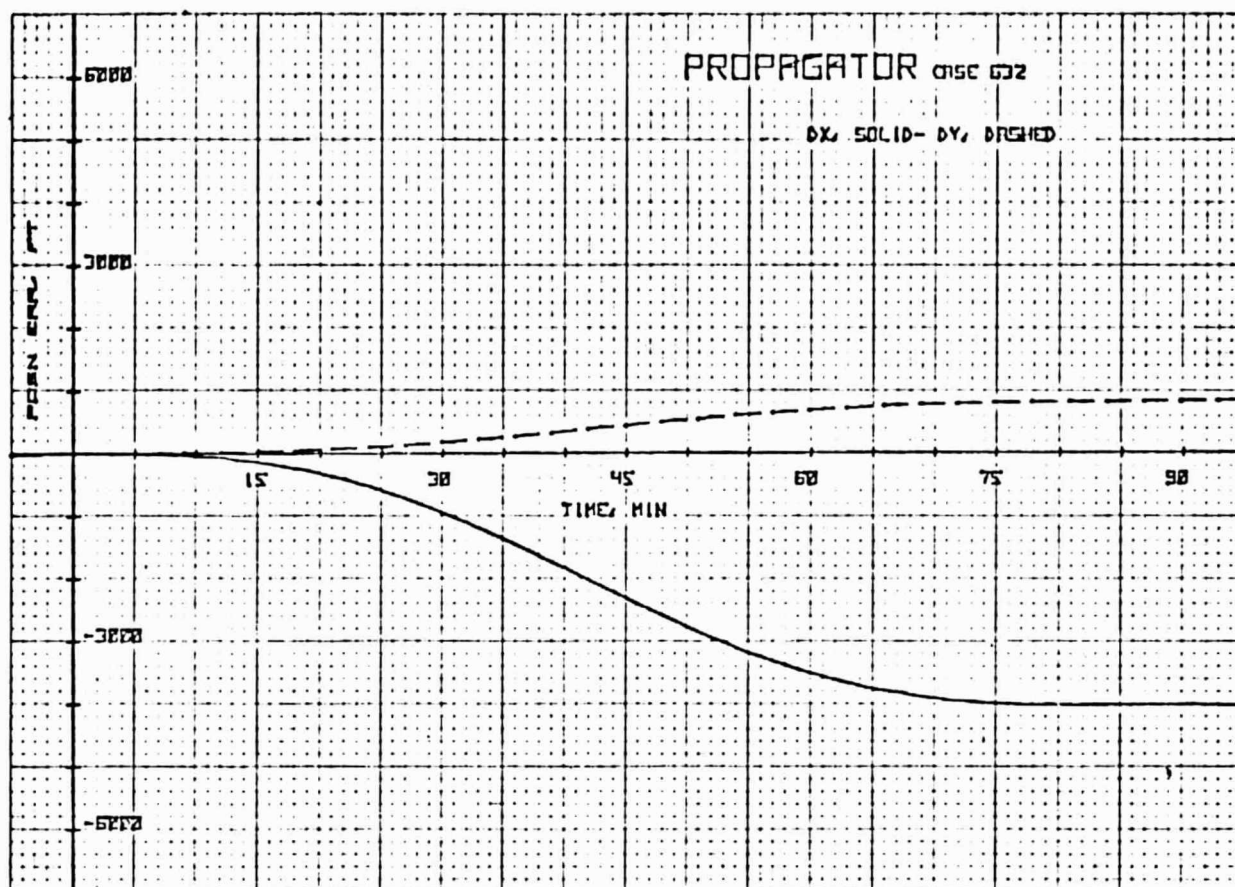
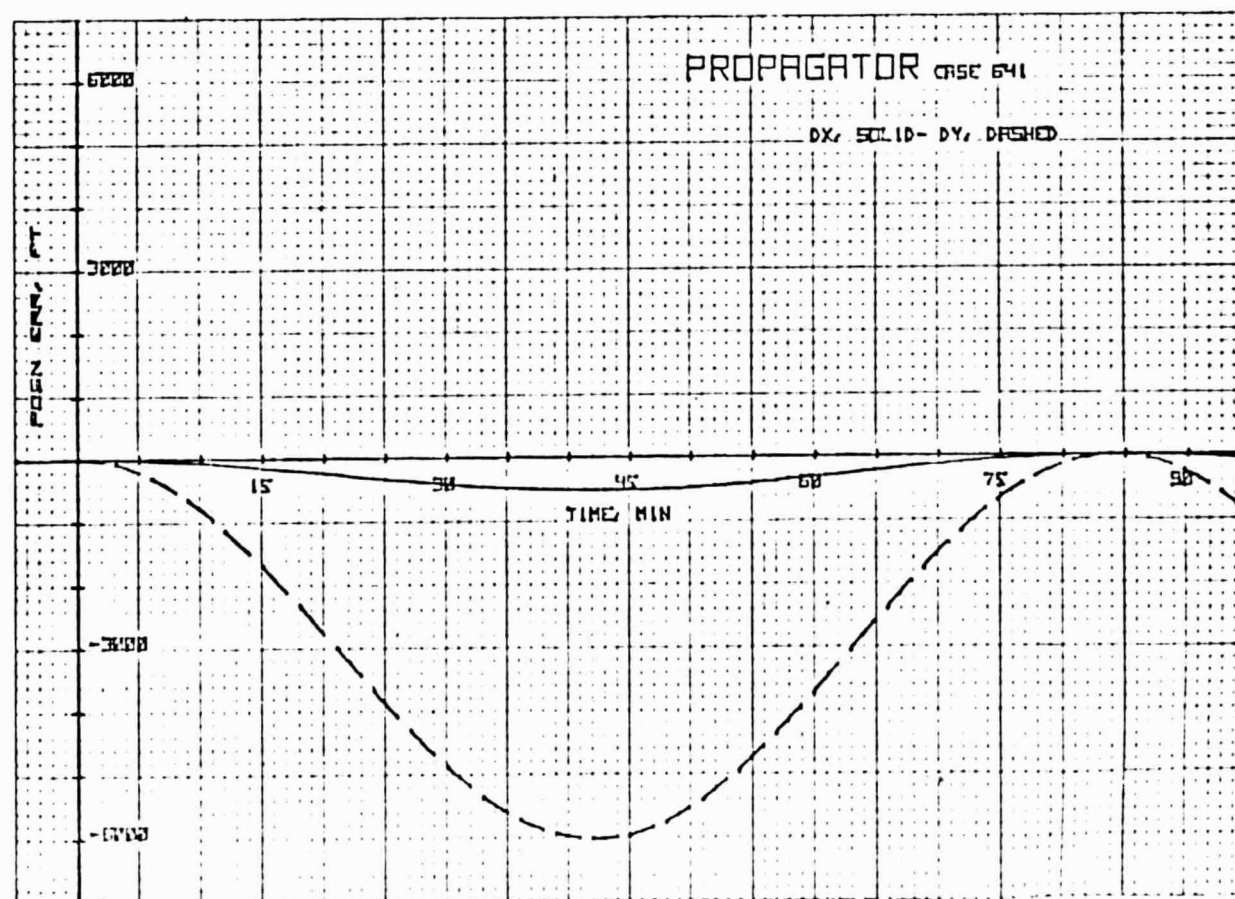


Figure 17. Case 632



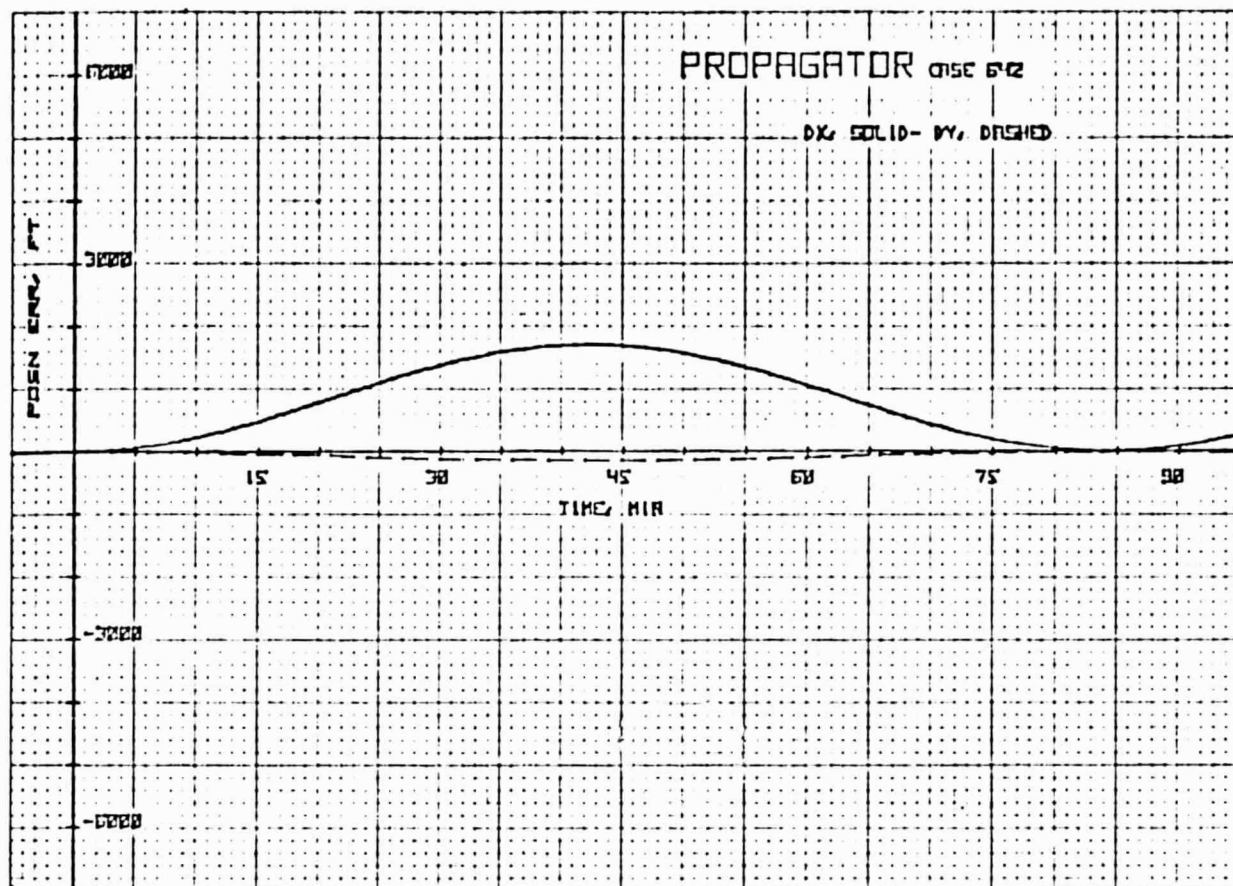


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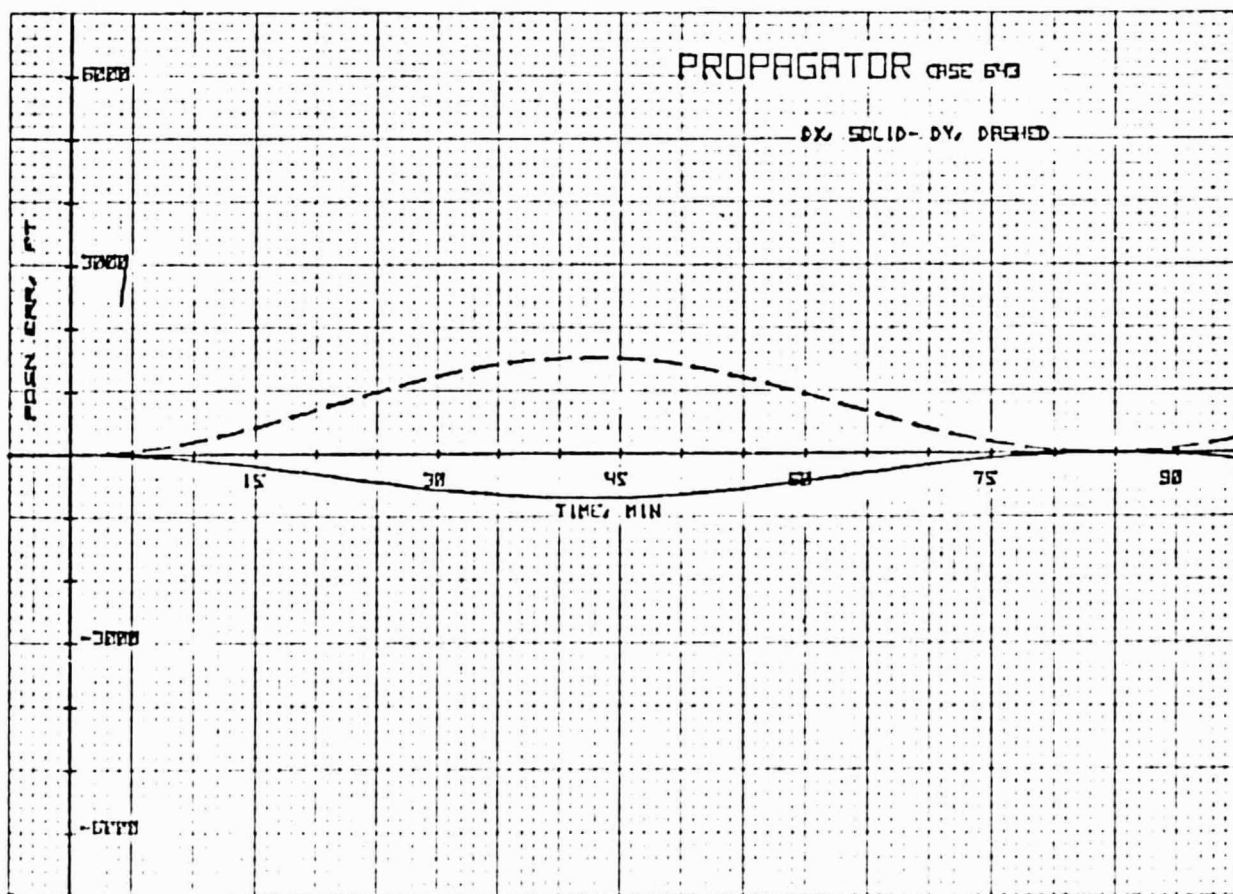


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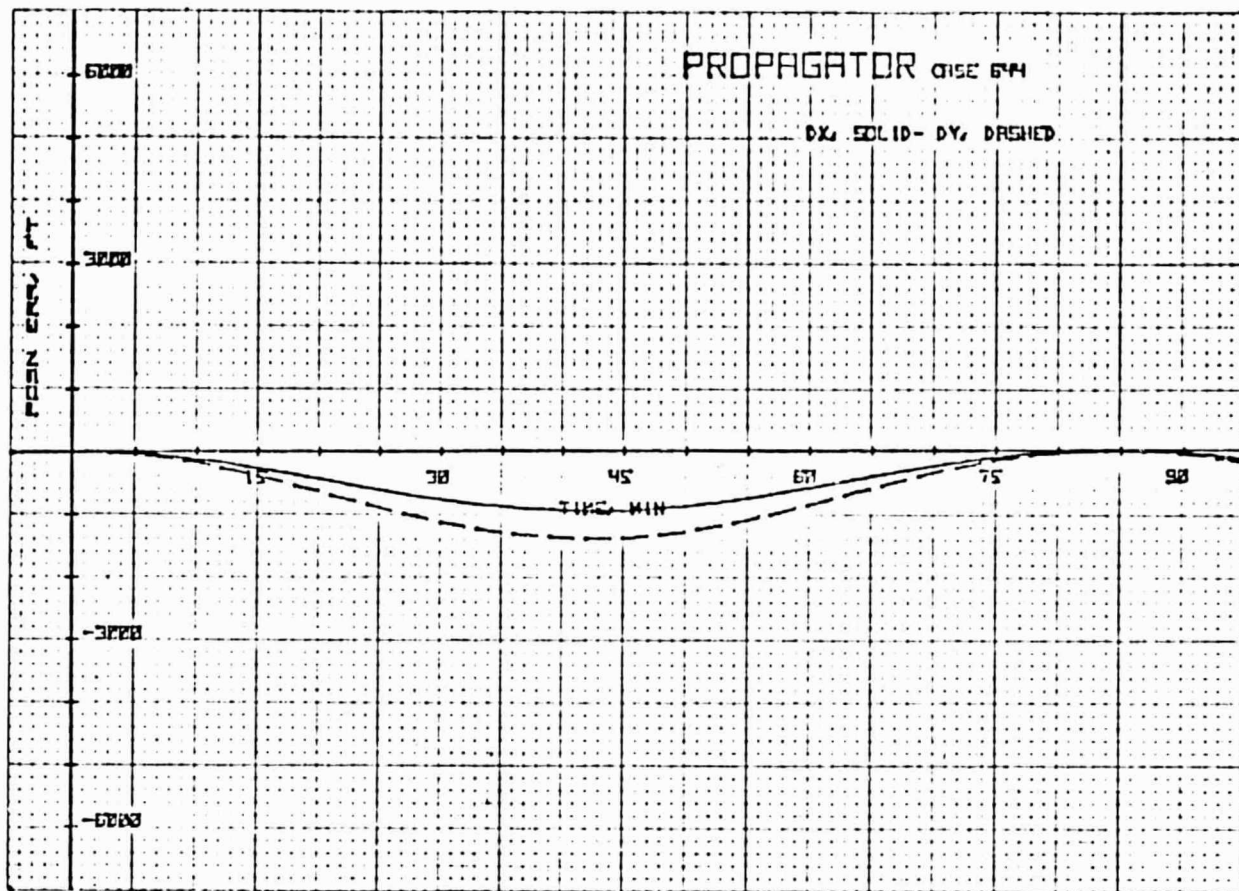


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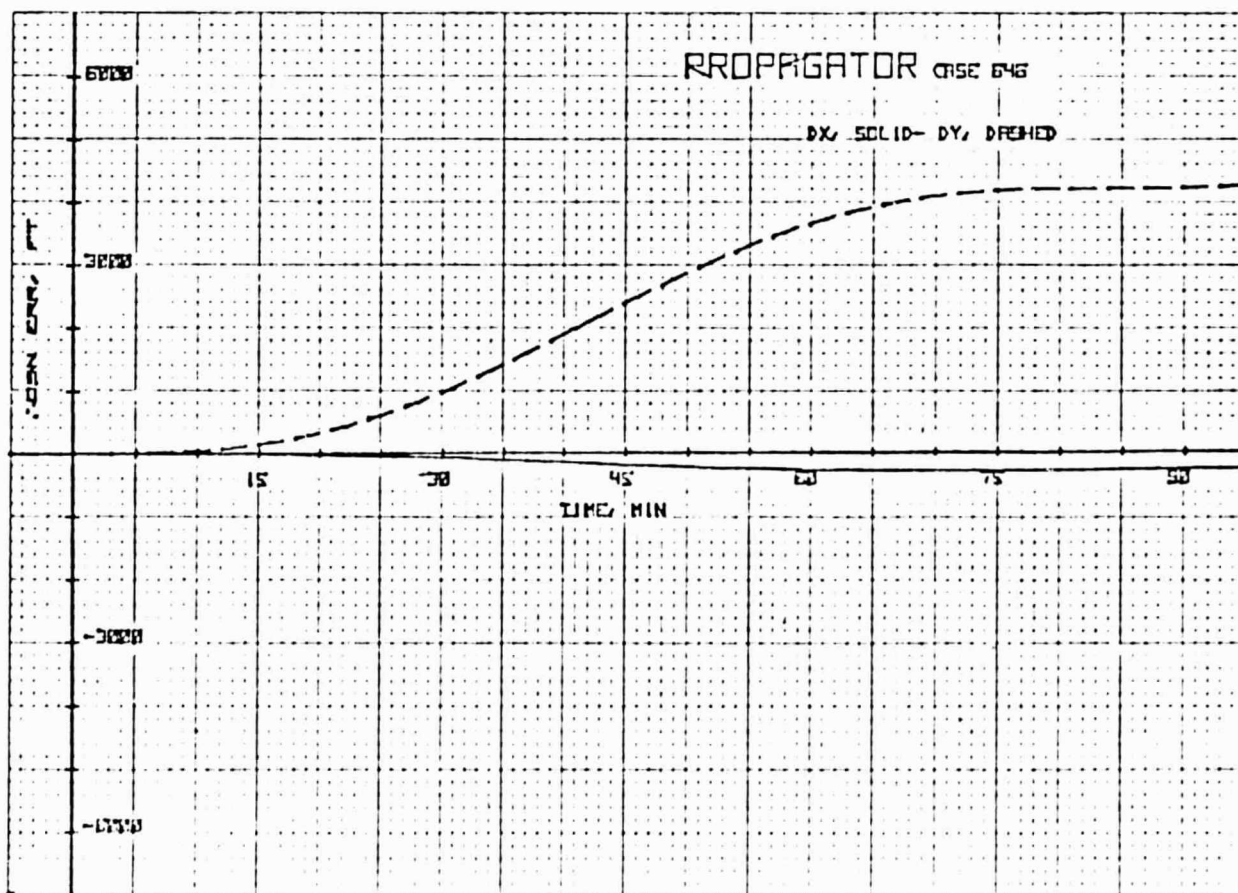


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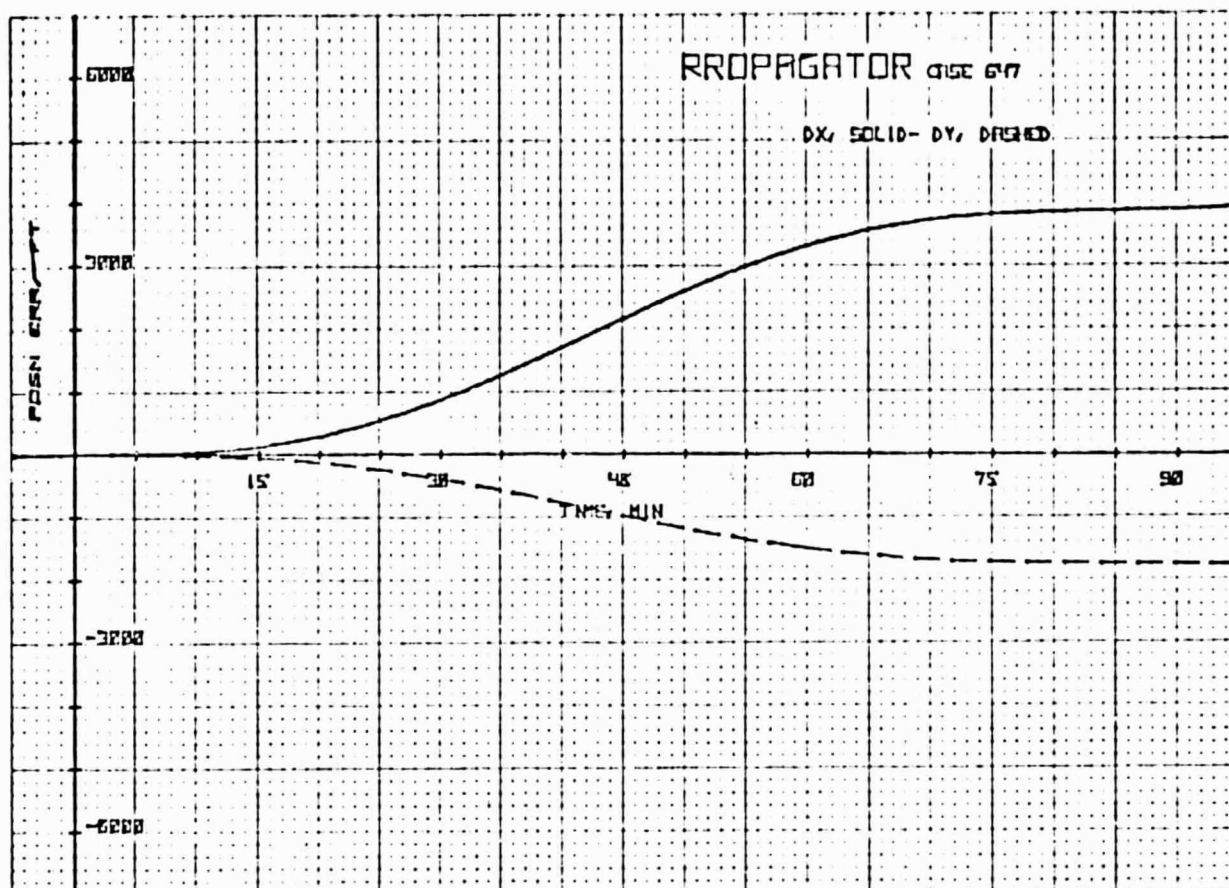


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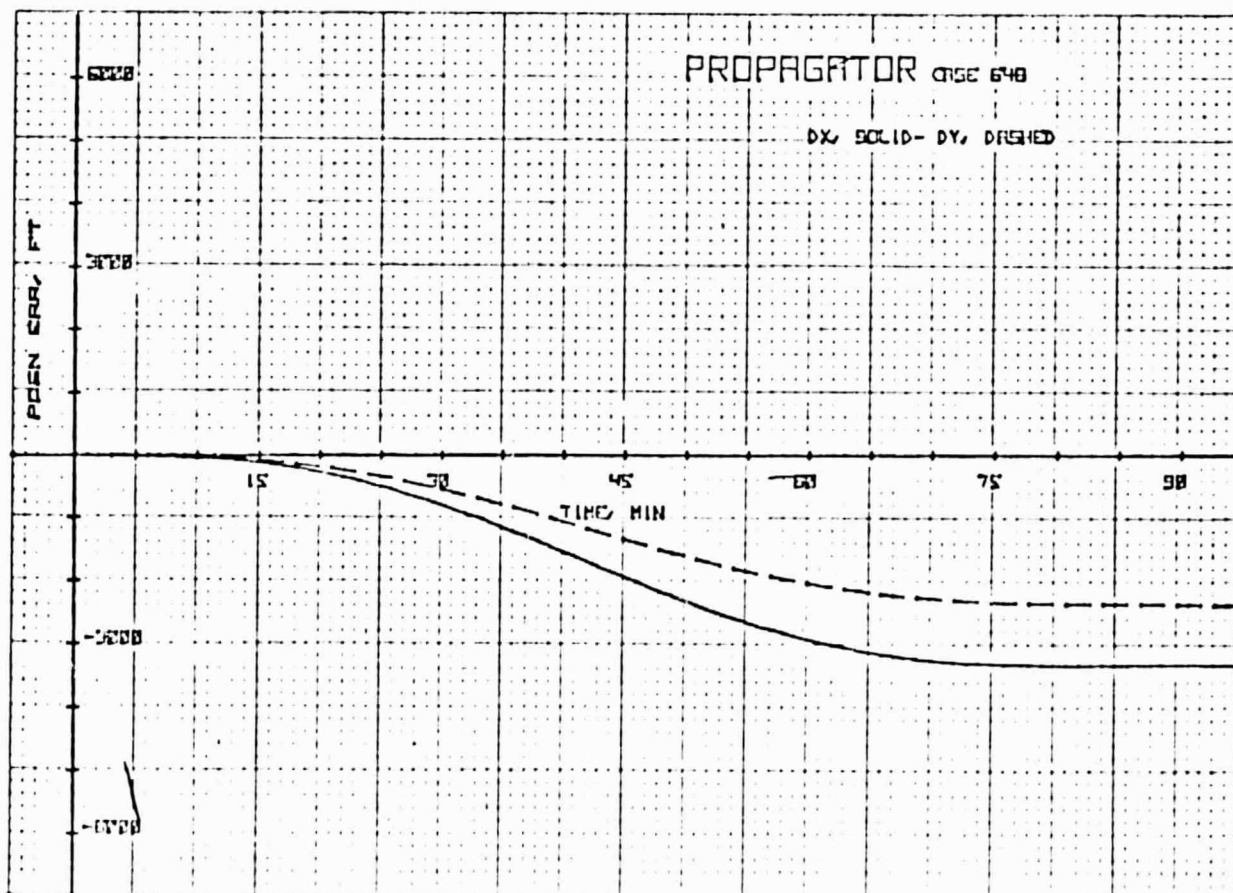


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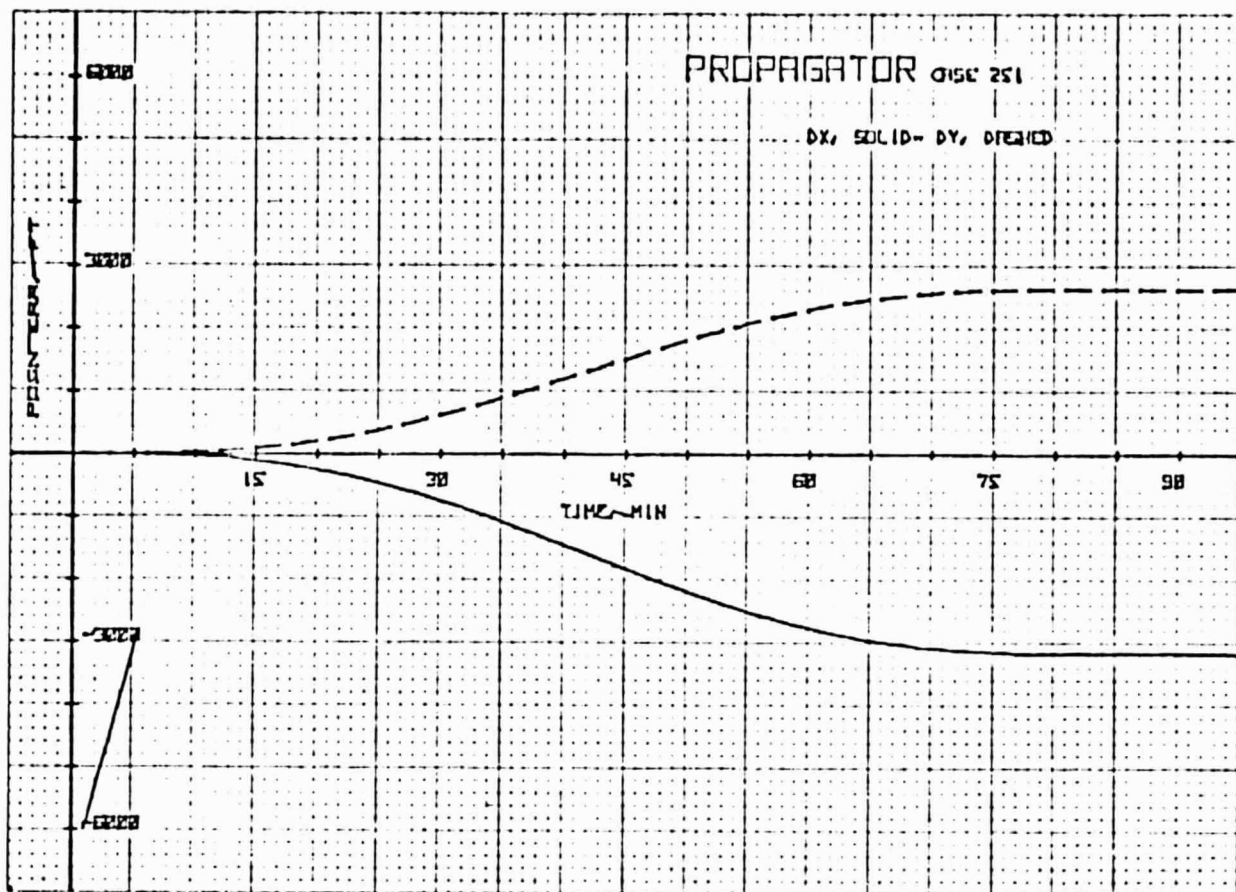
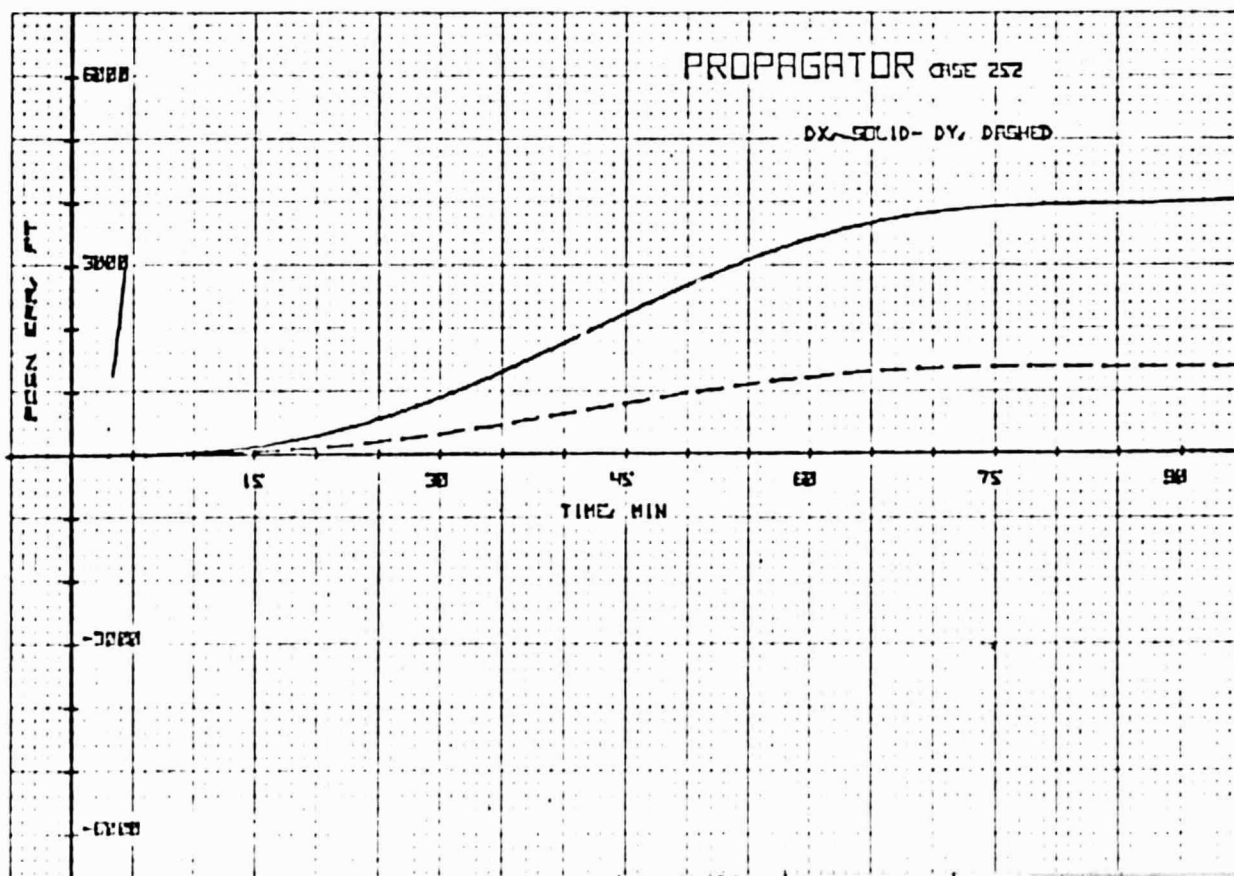


Figure 25. Case 251



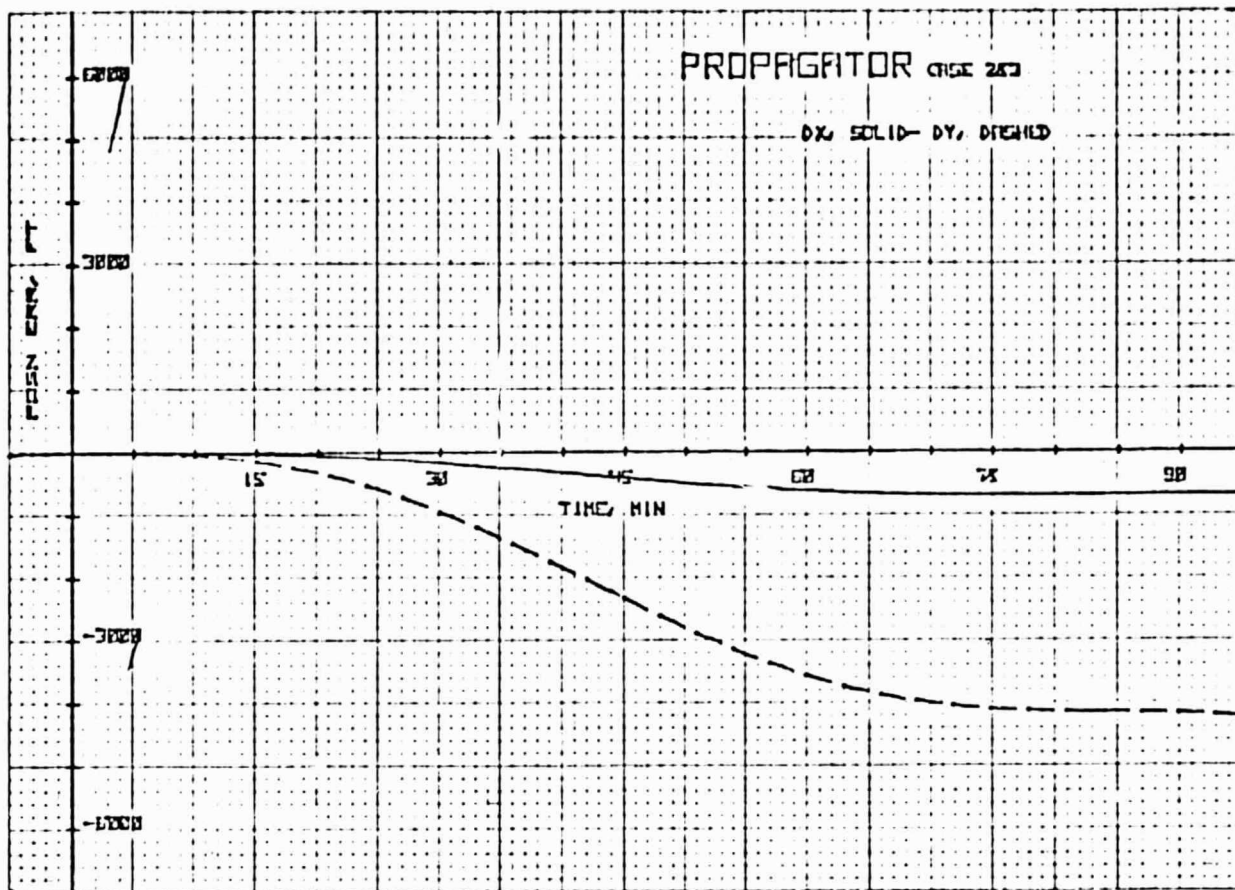


Figure 27. Case 283

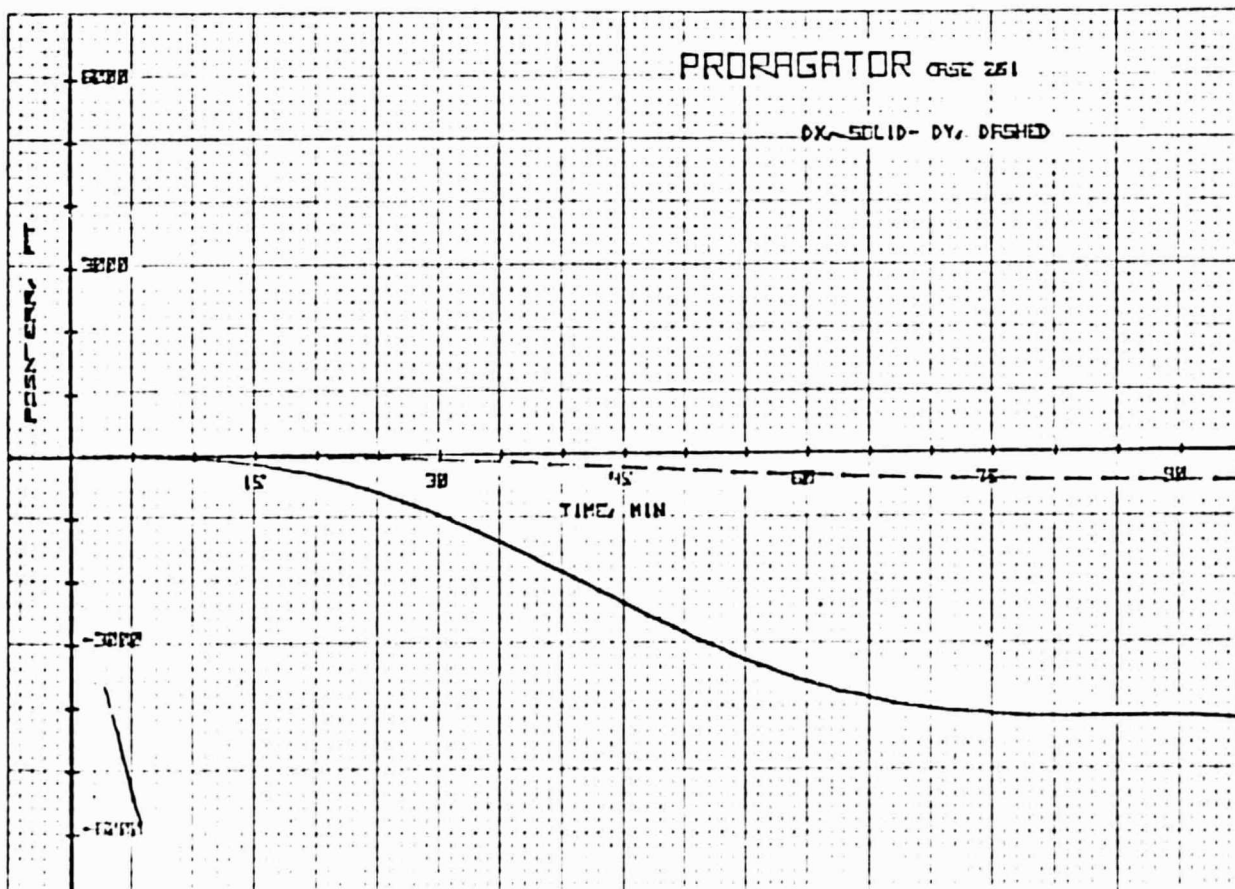


Figure 28. Case 281

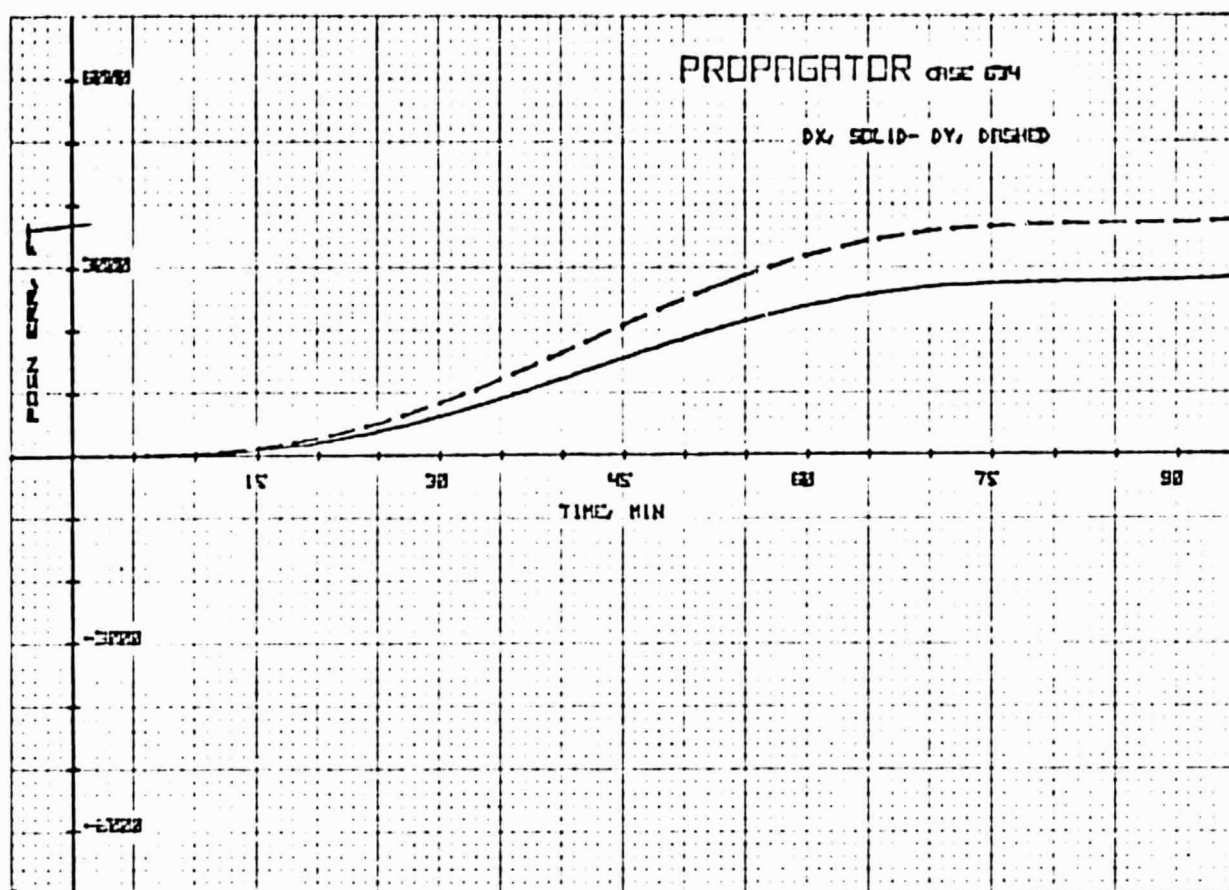
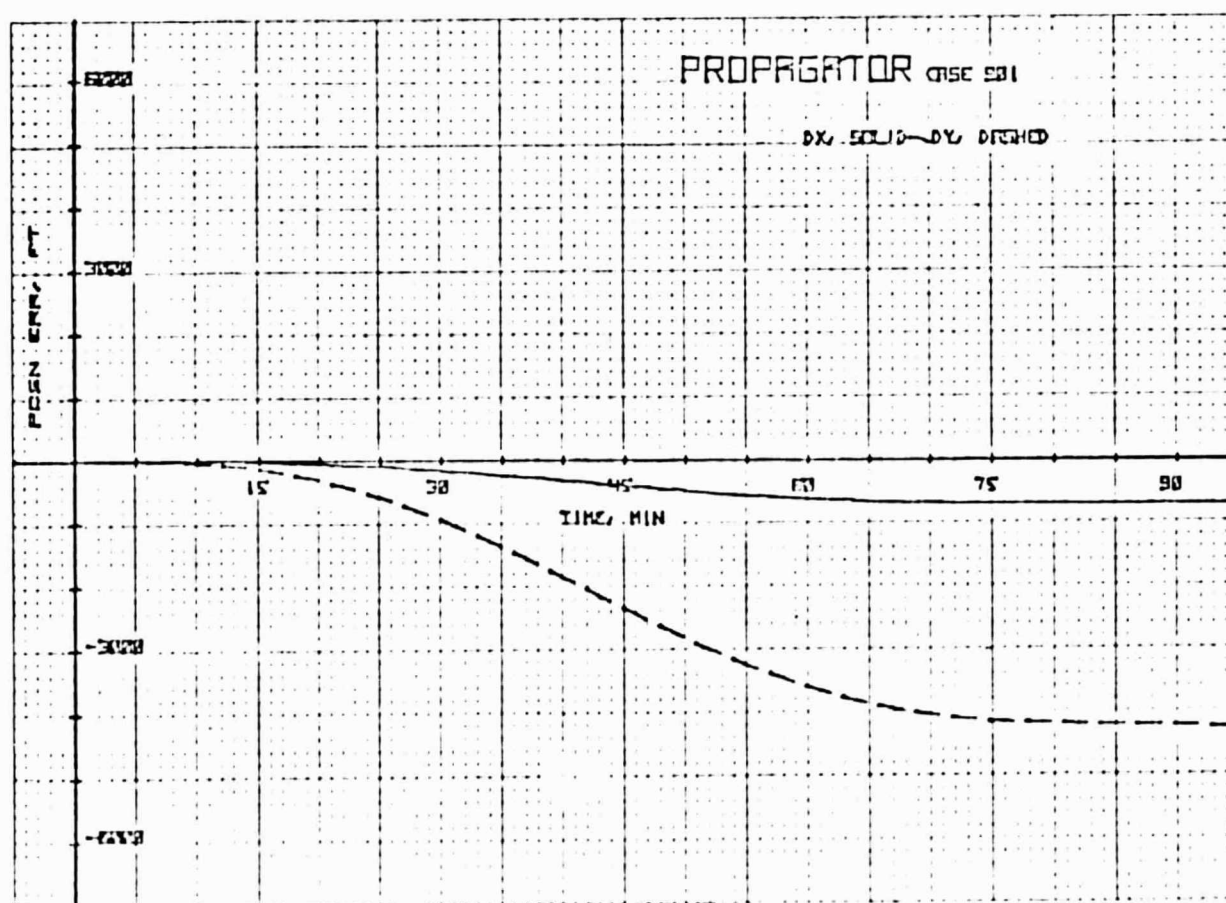


Figure 29. Case 634



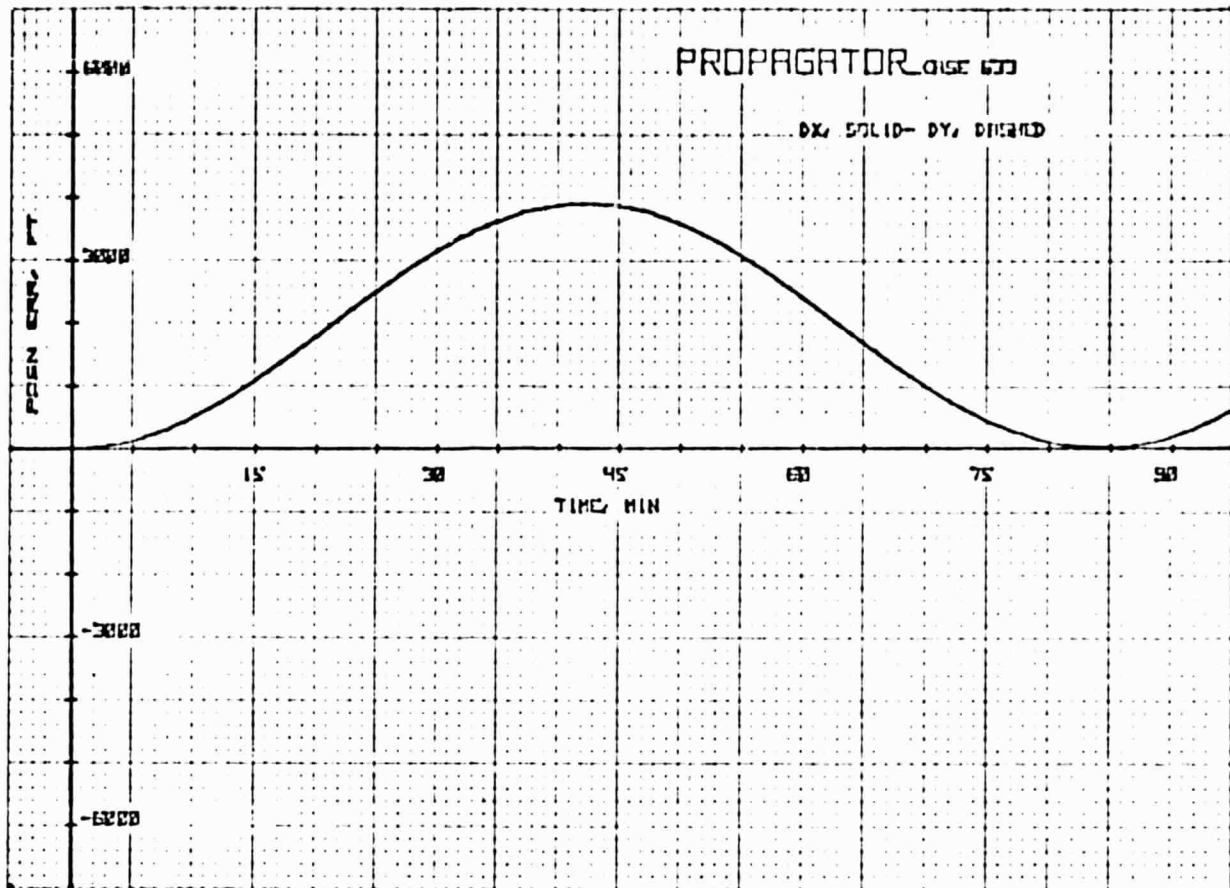


Figure 31. Case 633

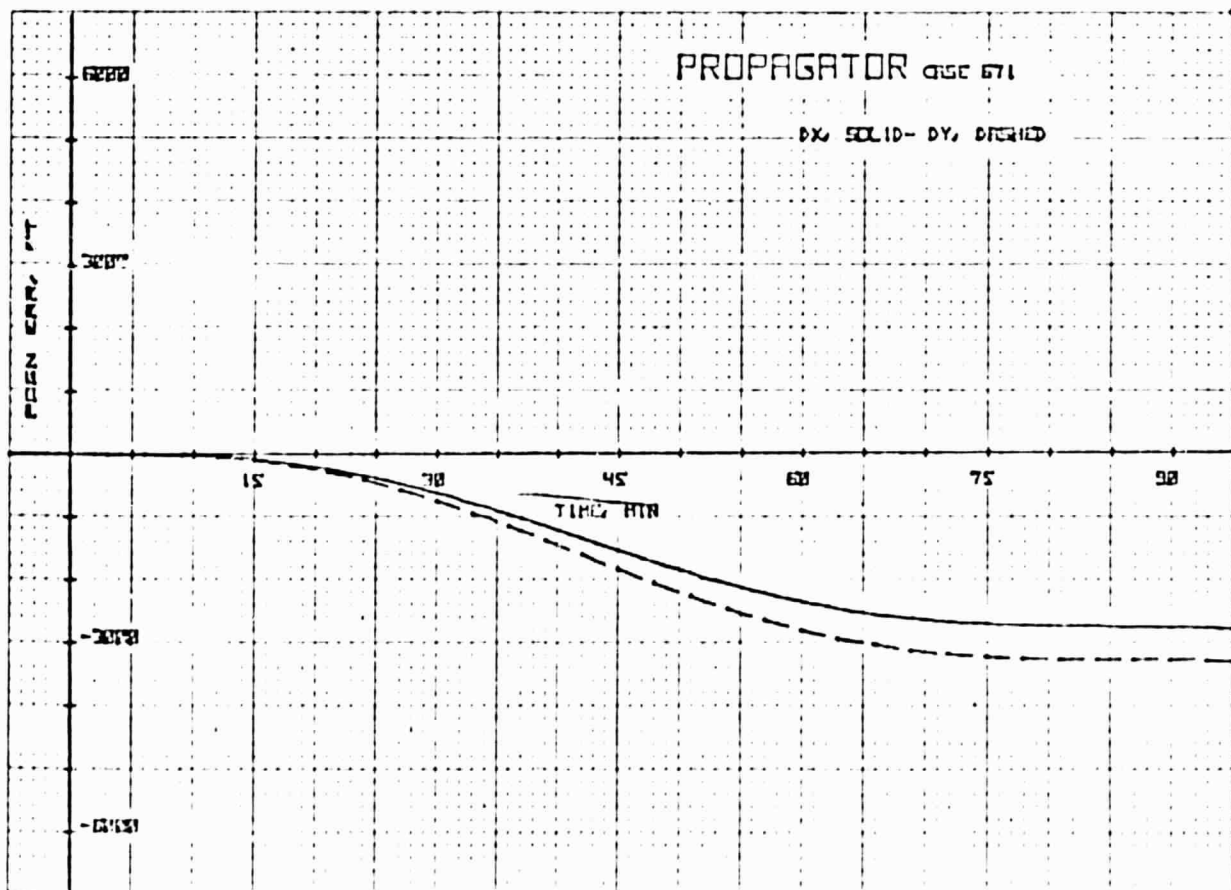


Figure 32. Case 671